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# Neutrino masses in the supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with right-handed neutrinos

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**Abstract.** The $R$-symmetry formalism is applied for the supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) model with right-handed neutrinos. For this kind of models, we study the generalization of the MSSM relation among $R$-parity, spin and matter parity. Discrete symmetries for the proton stable in this model are imposed, and we show that in such a case it is able to give leptons masses at only the tree level contributions required. A simple mechanism for the mass generation of the neutrinos is explored. We show that at the low-energy effective theory, the neutrino spectrum contains three Dirac fermions, one massless and two degenerate in mass. At the energy level where the mixing among them with the neutralinos is turned on, neutrinos obtain Majorana masses and correct the low-energy effective result which naturally gives rise to an inverted hierarchy mass pattern. This mass spectrum can fit the current data with minor fine-tuning. Consistent values for masses of the charged leptons are also given. In this model, the MSSM neutralinos and charginos can be explicitly identified in terms of the new constraints on masses which is not as in a supersymmetric version of the minimal 3-3-1 model.

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## 1 Introduction

Although the standard model (SM) gives very good results in explaining the observed properties of the charged fermions, it is unlikely to be the ultimate theory. It maintains the masslessness of the neutrinos to all orders in perturbation theory and even after non-pertubative effects are included. The recent groundbreaking discovery of nonzero neutrino masses and oscillations [1–7] has put massive neutrinos as one piece of evidence on physics beyond the SM.

The Super-Kamiokande experiments on atmospheric neutrino oscillations have indicated the difference of the squared masses and the mixing angle with fair accuracy [8, 9]:

$$\Delta m^2_{\mathrm{atm}} = 1.3 \div 3.0 \times 10^{-3}\,\mathrm{eV}^2\,, \tag{1}$$

$$\sin^2 2\theta_{\mathrm{atm}} > 0.9\,, \tag{2}$$

while those from the combined fit of the solar and reactor neutrino data point to

$$\Delta m^2_{\odot} = 8.0^{+0.6}_{-0.4} \times 10^{-5}\mathrm{eV}^2\,, \tag{3}$$

$$\tan^2\theta_{\odot} = 0.45^{+0.09}_{-0.07}\,. \tag{4}$$

Since the data provide only information about the differences in $m^2_\nu$, the neutrino mass pattern can be either almost degenerate or hierarchical. Among the hierarchical possibilities, there are two types of normal and inverted hierarchies. In the literature, in most of the cases one explores the normal hierarchical one. In this paper, we will comment on a supersymmetric model which naturally gives rise to three pseudo-Dirac neutrinos with an inverted hierarchical mass pattern.

The gauge symmetry of the SM as well as those of many extensional models by themselves fix only the gauge bosons. The fermions and Higgs contents have to be chosen somewhat arbitrarily. In the SM, these choices are made in such a way that the neutrinos are massless as mentioned. However, there are other choices based on the SM symmetry such that neutrinos become massive. We know these from the popular seesaw [10–14] and radiative [15–26] models. Particularly, the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge unification group [27–39], called 3-3-1 models, give stricter fermion contents. Indeed, only

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three fermion generations are acquired as a result of the anomaly cancellation and the condition of QCD asymptotic freedom. The arbitrariness in this case is only behind which SM singlets are put in the bottoms of the lepton triplets. In some scenarios, exotic leptons may exist in the singlets. A result of this is quite similar to the case of the SM neutrinos. As a fact, radiative mechanisms for neutrino masses arise, which been explored in [40–48].

Forbidding the exotic leptons, there are two main versions of the 3-3-1 models as far as minimal lepton sectors are concerned. In one of them [27–29] the three known left-handed lepton components for each generation are associated to three $SU(3)_L$ triplets by $(\nu_l, l, l^c)_L$, in which $l^c_L$ is related to the right-handed isospin singlet of the charged lepton $l$ in the SM. No extra leptons are needed and therefore it is called a minimal 3-3-1 model. In the variant model [30–34] the three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l, \nu^c_l)_L$, where $\nu^c_l$ is related to the right-handed component of the neutrino field $\nu_l$. These left-handed antineutrinos serve in the role of charge conjugation of the usual right-handed neutrinos which are required in the usual see-saw mechanism. We therefore call this a model with right-handed neutrinos. This kind of 3-3-1 models requires only a more economical Higgs sector for breaking the gauge symmetry and generating the fermion masses. Among the new gauge bosons in this model, the neutral non-Hermitian bilepton field $X^0$ may have a promising signature in accelerator experiments and may be also the source of neutrino oscillations [49–51]. In the current paper, the neutrinos of the 3-3-1 model with right-handed neutrinos are the subject for an extended study.

In the model considered, the Yukawa sector has automatic lepton-number conservation, which is broken only in the Higgs sector. At tree level, the neutrino spectrum contains three Dirac fermions, one massless and two degenerate in mass. This is clearly not realistic in view of the experimental data. However, this pattern may be severely changed by quantum corrections with the help of lepton-number violating Higgs couplings [52]. The neutrinos will obtain Majorana masses, which involve a new physics $SU(3)_L$ breaking scale and correct the tree-level result. This gives rise to two possible scenarios, depending on the size of such scale: (a) an usual see-saw mechanism if the $SU(3)_L$ breaking scale is very high; (b) an inverted hierarchy mass pattern (the tree level Dirac mass is of order $\Delta m_{\mathrm{atm}}$) if the $SU(3)_L$ breaking scale is not much higher than the weak scale. This scenario gives rise to a pseudo-Dirac neutrino mass pattern. This is a *specific* feature of the 3-3-1 model with right-handed neutrinos which was considered in [52] (see also [53]), but such effects exist in a very high level of loop corrections.

Some years ago, one of us has proposed the construction of the supersymmetric 3-3-1 model with right-handed neutrinos [54, 55]. In this paper, in the frameworks of the considered supersymmetric version, we explore a consistent neutrino mass spectrum of such a type but from a different side. Namely, the neutral fermion spectrum (neutrinos as well neutralinos) in this model contains the mentioned neutrinos. At tree level, the neutrino sector obtains a mass matrix just as the non-supersymmetric one. However, that tree-level neutrino sector mixes with the neutralinos in the supersymmetric version of the model [54, 55]. This will give real massive neutrinos.

In this case, we show that an inverted hierarchy mass spectrum for the neutrinos may be obtained, but only at the required tree level contribution which can fit the current data with some minor fine-tuning. Thus, our result differs from many extensions of the SM. As far as the mechanism is concerned, it obviously keeps to the kind of a seesaw one. It is not as in the case of the minimal 3-3-1 model, in which its supersymmetric version [56] gives only the real lepton mass spectra when the one loop corrections are added [57]. Moreover, in our case the charged leptons always gain consistent masses from different impacts due to mixing among the neutrinos with the neutralinos.

The outline of this work is as follows. In Sect. 2 we review the concept of $R$-symmetry and $R$-parity, in order to apply this concept on the supersymmetric 3-3-1 model with right-handed neutrinos. In Sect. 3 we define the $R$-charge in our model in order to get similar results as in the minimal supersymmetric standard model (MSSM). In Sect. 3.1 we impose another discrete symmetry that allows for neutrino masses but forbids proton decay. In Sect. 4 we calculate the fermion masses in our model. Our conclusions are found in the last section. Finally, in Appendix , we present the mass matrix elements of the neutral fermions.

## 2 $R$-symmetry

It is important to note that the SM can explain the conservation of lepton number ($L$) and of baryon number ($B$) without needing any discrete symmetry. However, this is not the case for supersymmetric theories, where only if interactions of the type conserving both $L$ and $B$ are required, one has to impose one discrete symmetry. This section recalls how $R$-parity emerged as a discrete remnant of continuous $U(1)$ $R$-symmetry which is necessarily broken so that the gauginos and gluinos acquire masses in the MSSM.

### 2.1 $R$-symmetry in superspace formalism

The $R$-symmetry was introduced in 1975 by Salam and Strathdee [58] and in an independent way by Fayet [59] to avoid the interactions that violate either lepton number or baryon number. There is very nice review of this subject in [60, 61].

The concept of $R$-symmetry is better understood in the superspace formalism, where the $R$-symmetry is a $U(1)$ continuous symmetry, parametrized by $\alpha$. The operator which produces this symmetry is going to be denoted as $\mathbf{R}$. This operator acts on the superspace coordinate $\theta, \bar{\theta}$ as follows [62]:

$$\begin{aligned}\mathbf{R}\theta &\to \mathrm{e}^{-\mathrm{i}\alpha}\theta\,,\\ \mathbf{R}\bar{\theta} &\to \mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}\,.\end{aligned} \tag{5}$$

Hence the $\theta$ has $R$-charge where $R(\theta) = -1$, while $\bar{\theta}$ is $R(\bar{\theta}) = 1$.

The operator $\mathbf{R}$ acts on chiral superfields $\Phi(x,\theta,\bar{\theta})$ and antichiral superfields $\bar{\Phi}(x,\theta,\bar{\theta})$, respectively, in the following way [63]:

$$\mathbf{R}\Phi(x,\theta,\bar{\theta}) = \mathrm{e}^{2in_\Phi\alpha}\Phi(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}) , \qquad (6)$$
$$\mathbf{R}\bar{\Phi}(x,\theta,\bar{\theta}) = \mathrm{e}^{-2in_\Phi\alpha}\bar{\Phi}(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}) , \qquad (7)$$

where $2n_\Phi$ is the $R$-charge of the above chiral superfield. This new charge $n_\Phi$ is an additive conserved quantum number. This operator acts on the vectorial superfield by the rule

$$\mathbf{R}V(x,\theta,\bar{\theta}) = V(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}) . \qquad (8)$$

The expansion of the superfields in terms of $\theta$ and $\bar{\theta}$ [63] is given by

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \\ &\quad + \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m A(x) - \frac{\mathrm{i}}{\sqrt{2}}(\theta\theta)\partial_m\psi(x)\sigma^m\bar{\theta} \\ &\quad + \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box A(x) , \qquad (9)\end{aligned}$$

$$\begin{aligned}\bar{\Phi}(x,\theta,\bar{\theta}) &= \bar{A}(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) + \bar{\theta}\bar{\theta}\bar{F}(x) \\ &\quad - \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m\bar{A}(x) + \frac{\mathrm{i}}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^m\partial_m\bar{\psi}(x) \\ &\quad + \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box\bar{A}(x) , \qquad (10)\end{aligned}$$

$$\begin{aligned}V_{WZ}(x,\theta,\bar{\theta}) &= -\theta\sigma^m\bar{\theta}A_m(x) + \mathrm{i}(\theta\theta)\bar{\theta}\bar{\lambda}(x) - \mathrm{i}(\bar{\theta}\bar{\theta})\theta\lambda(x) \\ &\quad + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x) \\ &\quad \text{(expansion in Wess–Zumino gauge)} , \qquad (11)\end{aligned}$$

where $A(x), F(x)$ and $D(x)$ are scalar fields; $\psi(x)$ and $\lambda(x)$ are fermion fields, while $A_m(x)$ is a vector field.

Combining (6) and (9) we get the following transformations for the field components, respectively:

$$\left.\begin{array}{l} A(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2in_\Phi\alpha}A(x) \\ \psi(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2\mathrm{i}\left(n_\Phi - \frac{1}{2}\right)\alpha}\psi(x) \\ F(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2\mathrm{i}(n_\Phi - 1)\alpha}F(x) \end{array}\right\} . \qquad (12)$$

Similarly, for the antichiral superfield we get

$$\left.\begin{array}{l} \bar{A}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-2in_\Phi\alpha}\bar{A}(x) \\ \bar{\psi}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-2\mathrm{i}\left(n_\Phi - \frac{1}{2}\right)\alpha}\bar{\psi}(x) \\ \bar{F}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-2\mathrm{i}(n_\Phi - 1)\alpha}\bar{F}(x) \end{array}\right\} . \qquad (13)$$

From (8) and (11), the field components in the vector superfield transform as

$$\left.\begin{array}{l} A_m(x) \xmapsto{\mathbf{R}} A_m(x) \\ \lambda(x) \xmapsto{\mathbf{R}} \mathrm{e}^{\mathrm{i}\alpha}\lambda(x) \\ \bar{\lambda}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-\mathrm{i}\alpha}\bar{\lambda}(x) \\ D(x) \xmapsto{\mathbf{R}} D(x) \end{array}\right\} . \qquad (14)$$

The transformations in (12), (13) and (14) can be rewritten in terms of 4-component spinors as [61]

$$\begin{array}{ll} & A(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2in_\Phi\alpha}A(x) , \\ A_m(x) \xmapsto{\mathbf{R}} A_m(x) , & \bar{A}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-2in_\Phi\alpha}\bar{A}(x) , \\ \Lambda(x) \xmapsto{\mathbf{R}} \mathrm{e}^{\mathrm{i}\gamma_5\alpha}\Lambda(x) , & \Psi(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2\mathrm{i}\gamma_5(n_\Phi - 1/2)\alpha}\Psi(x) , \\ D(x) \xmapsto{\mathbf{R}} D(x) , & F(x) \xmapsto{\mathbf{R}} \mathrm{e}^{2\mathrm{i}(n_\Phi - 1)\alpha}F(x) , \\ & \bar{F}(x) \xmapsto{\mathbf{R}} \mathrm{e}^{-2\mathrm{i}(n_\Phi - 1)\alpha}\bar{F}(x) . \end{array} \qquad (15)$$

In (15), $\Lambda$ is the Majorana spinor which represents the gauginos, while $\Psi(x)$ represents the Dirac spinor for quarks and leptons.

For products of left-handed chiral superfields, it is to be noted that

$$\mathbf{R}\prod_a \Phi_a(x,\theta,\bar{\theta}) = \mathrm{e}^{2\mathrm{i}\sum_a n_a\alpha}\prod_a \Phi_a(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta}) .$$

Thus, the general superfield terms given now,

$$\begin{aligned}&\int \mathrm{d}^4\theta\, \bar{\Phi}(x,\theta,\bar{\theta})\Phi(x,\theta,\bar{\theta}) , \\ &\int \mathrm{d}^4\theta\, \bar{\Phi}(x,\theta,\bar{\theta})\mathrm{e}^{V(x,\theta,\bar{\theta})}\Phi(x,\theta,\bar{\theta}) , \\ &\int \mathrm{d}^2\theta\, \prod_a \Phi_a(x,\theta,\bar{\theta}) , \qquad \text{if } \sum_a n_a = 1 , \qquad (16)\end{aligned}$$

are all $R$-invariant.

### 2.2 Continuous $R$-symmetry in MSSM

In the MSSM [64] the left-handed fermions are in doublets, whereas the right-handed antifermions are in singlets: $\hat{L}_{\mathrm{L}} \sim (1,\mathbf{2},-1)$, $\hat{l}^c_{\mathrm{L}} \sim (1,\mathbf{1},2)$ and $\hat{Q}_{\mathrm{L}} \sim (3,\mathbf{2},1/3)$, $\hat{u}^c_{\mathrm{L}} \sim (3^*,\mathbf{1},-4/3)$, $\hat{d}^c_{\mathrm{L}} \sim (3^*,\mathbf{1},2/3)$. The Higgs bosons are in doublets, $\hat{H}_1 \sim (1,\mathbf{2},-1)$ and $\hat{H}_2 \sim (1,\mathbf{2},1)$. With these multiplets, the superpotential of the model is written as

$$\begin{aligned}W_2 &= \mu\epsilon\hat{H}_1\hat{H}_2 + \mu_{0a}\epsilon\hat{L}_{a\mathrm{L}}\hat{H}_2 , \\ W_3 &= f^l_{ab}\epsilon\hat{L}_{a\mathrm{L}}\hat{H}_1\hat{l}^c_{b\mathrm{L}} + f^u_{ij}\epsilon\hat{Q}_{i\mathrm{L}}\hat{H}_2\hat{u}^c_{j\mathrm{L}} + f^d_{ij}\epsilon\hat{Q}_{i\mathrm{L}}\hat{H}_1\hat{d}^c_{j\mathrm{L}} \\ &\quad + \lambda_{abc}\epsilon\hat{L}_{a\mathrm{L}}\hat{L}_{b\mathrm{L}}\hat{l}^c_{cL} + \lambda'_{iaj}\epsilon\hat{Q}_{i\mathrm{L}}\hat{L}_{a\mathrm{L}}\hat{d}^c_{j\mathrm{L}} \\ &\quad + \lambda''_{ijk}\hat{d}^c_{i\mathrm{L}}\hat{u}^c_{j\mathrm{L}}\hat{d}^c_{k\mathrm{L}} . \qquad (17)\end{aligned}$$

Hereafter, the superscript L will be removed from the superfields, and the $SU(2)$ indices are default. The superscript $^c$ indicates the charge conjugation and $\epsilon$ is the antisymmetric $SU(2)$ tensor. The sub-indices $a, b, c$ run over the lepton generations $e, \mu, \tau$, and $i, j, k = 1, 2, 3$ run over the quark ones.

Because of (6) and (7), the terms proportional to $\lambda, \lambda'$ and $\lambda''$ are forbidden by the $R$-symmetry. The following example illustrates this statement. Suppose that

$$\begin{aligned}&n_{H_1} = n_{H_2} = 0 , \ \text{for } H_1 , \ H_2 , \\ &n_Q = n_u = n_d = n_{\mathrm{L}} = n_l = \frac{1}{2} , \ \text{for } Q, u^c, \ d^c, \ L, \ l^c , \qquad (18)\end{aligned}$$

which imply

$$\hat{H}_{1,2}(x,\theta,\bar{\theta}) \stackrel{\mathbf{R}}{\longmapsto} \hat{H}_{1,2}(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta})\,, \quad (19)$$

$$\hat{\Phi}(x,\theta,\bar{\theta}) \stackrel{\mathbf{R}}{\longmapsto} \mathrm{e}^{\mathrm{i}\alpha}\,\hat{\Phi}(x,\mathrm{e}^{-\mathrm{i}\alpha}\theta,\mathrm{e}^{\mathrm{i}\alpha}\bar{\theta})\,,$$
$$\hat{\Phi} = Q,\ u^c,\ d^c,\ L,\ l^c\,. \quad (20)$$

By (12) and (13), their components transform as

$$\begin{aligned}
H_{1,2}(x) &\stackrel{\mathbf{R}}{\longmapsto} H_{1,2}(x)\,,\\
\tilde{H}_{1,2}(x) &\stackrel{\mathbf{R}}{\longmapsto} \mathrm{e}^{-\mathrm{i}\alpha}\,\tilde{H}_{1,2}(x)\,,\\
\tilde{f}_{\mathrm{L}}(x) &\stackrel{\mathbf{R}}{\longmapsto} \mathrm{e}^{\mathrm{i}\alpha}\,\tilde{f}_{\mathrm{L}}(x)\,,\\
\tilde{f}^c_{\mathrm{L}}(x) &\stackrel{\mathbf{R}}{\longmapsto} \mathrm{e}^{-\mathrm{i}\alpha}\,\tilde{f}^c_{\mathrm{L}}(x)\,,\\
\Psi(x) &\stackrel{\mathbf{R}}{\longmapsto} \Psi(x)\,.
\end{aligned} \quad (21)$$

We recall that $H_{1,2}(x)$ are the Higgs bosons, $\tilde{H}_{1,2}(x)$ are the higgsinos, $\tilde{f}$ are the squarks and sleptons, and $\Psi(x)$ are the quarks and leptons. The consequence of the above transformation is summarized as

$$\begin{aligned}
\text{ordinary particle} &\stackrel{\mathbf{R}}{\longmapsto} \text{ordinary particle}\,,\\
\text{supersymmetric partner} &\stackrel{\mathbf{R}}{\longmapsto} \mathrm{e}^{\pm\mathrm{i}\alpha}\text{supersymmetric partner}.
\end{aligned} \quad (22)$$

Under the transformation law in (21), the conserving terms are given by

$$W = f^l\hat{L}\hat{H}_1\hat{l}^c + f^d\hat{Q}\hat{H}_1\hat{d}^c + f^u\hat{Q}\hat{H}_2\hat{u}^c\,. \quad (23)$$

Therefore, the couplings $\lambda, \lambda'$ and $\lambda''$ are forbidden by the charge assignment given in (18). These terms, if they were allowed, would induce the rapid proton decay. We allow only the terms from which the fermions in the model gain masses [60].

### 2.3 Problem with continuous $R$-symmetry, discrete $R$-parity

Because of (16), all the Lagrangians are invariant under the continuous $R$-symmetry and this obviously avoids the proton decay. However, such an unbroken continuous $R$-symmetry which acts on the gaugino and gluino mass terms would keep them massless, even after spontaneous breaking of the supersymmetry. To see this, let us remember that the gaugino's mass term is given by [65]

$$m_\lambda\left(\lambda\lambda + \bar{\lambda}\bar{\lambda}\right)\,, \quad (24)$$

which, under the $R$-symmetry (14), transforms as

$$m_\lambda\left(\mathrm{e}^{2\mathrm{i}\alpha}\lambda\lambda + \mathrm{e}^{-2\mathrm{i}\alpha}\bar{\lambda}\bar{\lambda}\right)\,. \quad (25)$$

As a result, the mass term (24) is not invariant under the $R$-symmetry. This fact forces us to abandon the continuous $R$-symmetry, in favour of the discrete $R$-symmetry, called *$R$-parity*. Thereby the $R$-parity automatically allows for gluino and other gaugino masses.

However, there is another possibility to guarantee $R$-parity conservation in the MSSM. It was firstly shown that in the model where $B$–$L$ conservation is automatic [66], $R$-parity can be broken by the vacuum, leading to an effective theory at low energies that has induced $R$-parity violating interactions [67]. Here, in this article, we will not work in this direction and will return our considerations to $R$-parity conservation.

The discrete $R$-parity, denoted by $\mathbf{R}_{\mathrm{d}}$, which is able to solve the above problem can be obtained by putting $\alpha = \pi$. Taking this value into account in (5), (6), (7) and (8) we get the following transformations:

$$\begin{aligned}
\mathbf{R}_{\mathrm{d}}\theta &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} -\theta,\\
\mathbf{R}_{\mathrm{d}}\bar{\theta} &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} -\bar{\theta}\,,\\
\mathbf{R}_{\mathrm{d}}\Phi(x,\theta,\bar{\theta}) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} \mathrm{e}^{2\mathrm{i}n_\Phi\pi}\Phi(x,-\theta,-\bar{\theta})\,,\\
\mathbf{R}_{\mathrm{d}}\bar{\Phi}(x,\theta,\bar{\theta}) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} \mathrm{e}^{-2\mathrm{i}n_\Phi\pi}\bar{\Phi}(x,-\theta,-\bar{\theta})\,,\\
\mathbf{R}_{\mathrm{d}}V(x,\theta,\bar{\theta}) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} V(x,-\theta,-\bar{\theta})\,.
\end{aligned} \quad (26)$$

It is worth emphasizing that, under this (discrete) transformation law, the terms $\theta\theta$ and $\theta\theta\bar{\theta}\bar{\theta}$ are invariants, which is very helpful in our further analysis.

Now, under the discrete symmetry, the components of the superfields transform as

$$\left.\begin{aligned}
A(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} \mathrm{e}^{2\mathrm{i}n_\Phi\pi}A(x)\\
\psi(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} \mathrm{e}^{2\mathrm{i}\left(n_\Phi-\frac{1}{2}\right)\pi}\psi(x)\\
F(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} \mathrm{e}^{2\mathrm{i}(n_\Phi-1)\pi}F(x)
\end{aligned}\right\}, \quad (27)$$

$$\left.\begin{aligned}
A_m(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} A_m(x)\\
\lambda(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} -\lambda(x)\\
\bar{\lambda}(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} -\bar{\lambda}(x)\\
D(x) &\stackrel{\mathbf{R}_{\mathrm{d}}}{\longmapsto} D(x)
\end{aligned}\right\}. \quad (28)$$

From (28), we see that (24) is, of course, invariant under the discrete symmetry, as mentioned. Moreover, the last term in (16) can be redefined by

$$\int \mathrm{d}^2\theta\,\prod_a \Phi_a(x,\theta,\bar{\theta})\,, \qquad \text{if}\ \sum_a n_a = 0\,. \quad (29)$$

In the following, we will show that there is a close connection between $R$-parity and baryon- and lepton-number conservation laws. Its origin is our desire to get supersymmetric theories in which $B$ and $L$ could be conserved simultaneously avoiding unwanted exchanges of spin-0 particles.

### 2.4 Discrete $R$-Parity in MSSM

Applying the conditions coming from (29) on Lagrangians in (17) we get the following equations:

$$n_{H_1} + n_{H_2} = 0\,,\ n_{\mathrm{L}} + n_{H_2} = 0\,, \quad (30)$$
$$n_{H_1} + n_{\mathrm{L}} + n_l = 0\,,\ n_{H_1} + n_Q + n_d = 0\,, \quad (31)$$
$$n_{H_2} + n_Q + n_u = 0\,,\ n_Q + n_{\mathrm{L}} + n_d = 0\,, \quad (32)$$
$$2n_{\mathrm{L}} + n_l = 0\,,\ 2n_d + n_u = 0\,. \quad (33)$$

Unfortunately, not all of these relations can be satisfied simultaneously. Only some of these constraints can be satisfied. For example, choosing

$$n_{H_1} = 0\,, \; n_{H_2} = 0\,, \; n_L = \frac{1}{2}\,, \; n_Q = \frac{1}{2}\,,$$
$$n_l = -\frac{1}{2}\,, \; n_u = -\frac{1}{2}\,, \; n_d = -\frac{1}{2}\,, \tag{34}$$

the superfields will transform as

$$\hat{V}(x,\theta,\bar{\theta}) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} \hat{V}(x,-\theta,-\bar{\theta})\,, \tag{35}$$
$$\hat{H}_{1,2}(x,\theta,\bar{\theta}) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} \hat{H}_{1,2}(x,-\theta,-\bar{\theta})\,, \tag{36}$$
$$\hat{\Phi}(x,\theta,\bar{\theta}) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} -\hat{\Phi}(x,-\theta,-\bar{\theta}), \;\; \Phi = Q,\; u^c,\; d^c,\; L,\; l^c\,. \tag{37}$$

In terms of the field components, we obtain

$$A_m(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} A_m(x)\,,\; H_{1,2}(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} H_{1,2}(x)\,, \quad \tilde{f}(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} -\tilde{f}(x)\,,$$
$$\Lambda(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} -\Lambda(x)\,,\; \tilde{H}_{1,2}(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} -\tilde{H}_{1,2}(x)\,,\; \Psi(x) \overset{\mathbf{R}_\mathrm{d}}{\longmapsto} \Psi(x)\,. \tag{38}$$

Therefore, the first condition in (30), both conditions in (31), and again the first condition in (32) are satisfied, and this does not happen with the remaining conditions. The terms in the superpotential (17) which satisfy the rule with the parameters in (34) are

$$W = \mu\epsilon \hat{H}_1 \hat{H}_2 + f^l_{ab}\epsilon \hat{L}_a \hat{H}_1 \hat{l}^c_b + f^u_{ij}\epsilon \hat{Q}_i \hat{H}_2 \hat{u}^c_j + f^d_{ij}\epsilon \hat{Q}_i \hat{H}_1 \hat{d}^c_j\,. \tag{39}$$

Their others terms are forbidden which is the background that $\lambda$ and $\lambda'$ are kinds of lepton-number violating parameters, while $\lambda''$ is a type of baryon-number violating parameter.

Equation (38) suggests us to classify the particles into two types of so called $R$-even and $R$-odd type. Here the $R$-even particles ($R_\mathrm{d} = +1$) include the gluons, photon, $W^\pm$ and $Z$ gauge bosons, the quarks, the leptons and the Higgs bosons. On the contrary, the $R$-odd particles ($R_\mathrm{d} = -1$) are their superpartners, i.e., the gluinos, neutralinos, charginos, squarks and sleptons. Therefore, $R$-parity is parity of $R$-charge of the continuous $U(1)$ $R$-symmetry and defined by

$$R\text{-parity} = \begin{cases} +1 & \text{for ordinary particles,} \\ -1 & \text{for their superpartners.} \end{cases} \tag{40}$$

The above intimate connection between $R$-parity and baryon-number and lepton-number conservation laws can be made explicitly by re-expressing (40) in terms of the spin $S$ and the matter-parity $(-1)^{3B+L}$ as follows [69]:

$$R\text{-parity} = (-1)^{2S}(-1)^{3B+L}\,. \tag{41}$$

Therefore, all scalar fields ($S = 0$) can be assigned $R$ values.

1. Usual scalars: $B = L = 0 \Longrightarrow R = +1$.
2. Sleptons: $B = 0$, $L = 1 \Longrightarrow R = -1$.
3. Squarks: $B = \frac{1}{3}$, $L = 0 \Longrightarrow R = -1$.

Analogously for fermions ($S = 1/2$).

1. Gauginos: $B = L = 0 \Longrightarrow R = -1$.
2. Leptons: $B = 0,\; L = 1 \Longrightarrow R = +1$.
3. Quarks: $B = \frac{1}{3},\; L = 0 \Longrightarrow R = +1$.

Because of the gauge bosons as well as all vectorial fields having $B = L = 0$, $R$-parity $= +1$. It is to be noted that the above assignment is correct only for the MSSM, where the vector gauge bosons do not carry the lepton number ($L = 0$).

To finish this section, let us note that there will be a lot of other choices of the charges in (33) forbidding fast proton decay [70]. However, all such choices are due to the action of $R$-symmetry which in a general way can be written as [71]

$$\Phi \longrightarrow \mathrm{e}^{2\mathrm{i}n_\Phi \frac{2\pi}{N}}\Phi\,. \tag{42}$$

Here it is similar to a $Z_N$ symmetry. Among those choices, there is a possibility which allows neutrinos to gain masses. Indeed, choosing

$$n_{H_1} = n_{H_2} = n_\mathrm{L} = n_l = 0\,,$$
$$n_Q = \frac{1}{2}\,,\; n_u = n_d = -\frac{1}{2}\,, \tag{43}$$

we get the following transformation of the superfields:

$$(Q, u^c, d^c) \to -(Q, u^c, d^c)\,,$$
$$(L, l^c, H_1, H_2) \to (L, l^c, H_1, H_2)\,. \tag{44}$$

The terms which are allowed by this new $R$-parity are obtained by

$$W = \mu\epsilon \hat{H}_1 \hat{H}_2 + \mu_{0a}\hat{L}_a \hat{H}_2 + f^l_{ab}\epsilon \hat{L}_a \hat{H}_1 \hat{l}^c_b + f^u_{ij}\epsilon \hat{Q}_i \hat{H}_2 \hat{u}^c_j$$
$$+ f^d_{ij}\epsilon \hat{Q}_i \hat{H}_1 \hat{d}^c_j + \lambda_{abc}\epsilon \hat{L}_a \hat{L}_b \hat{l}^c_c + \lambda'_{iaj}\epsilon \hat{Q}_i \hat{L}_a \hat{d}^c_j\,. \tag{45}$$

As shown in [72–74], this superpotential gives neutrinos masses.

## 3 Discrete $R$-parity in SUSY 3-3-1 RN

In the supersymmetric 3-3-1 model with right-handed neutrinos (SUSY 3-3-1 RN) [54, 55], the fermionic content is the following: the left-handed fermions are in triplets/antitriplets $L_a = (\nu_a, l_a, \nu^c_a)_\mathrm{L} \sim (\mathbf{1}, \mathbf{3}, -1/3)$, $a = e,\; \mu,\; \tau$; $Q_{\alpha L} = (d_\alpha, u_\alpha, d'_\alpha) \sim (\mathbf{3}, \mathbf{3}^*, 0)$, $\alpha = 1,\; 2$, $Q_{3L} = (u_3, d_3, u') \sim (\mathbf{3}, \mathbf{3}, 1/3)$. The right-handed components are in singlets: $l^c_a \sim (\mathbf{1}, \mathbf{1}, 1)$, $u^c_i$, $d^c_i$, $i = 1,\; 2,\; 3$, which are similar to those in the SM. In addition, the exotic quarks transform as $u'^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3)$, $d'^c_\alpha \sim (\mathbf{3}^*, \mathbf{1}, 1/3)$. The scalar content is minimally formed by three Higgs triplets: $\eta = (\eta^0_1, \eta^-, \eta^0_2)^\mathrm{T} \sim (\mathbf{1}, \mathbf{3}, -1/3)$; $\chi = (\chi^0_1, \chi^-, \chi^0_2)^\mathrm{T} \sim (\mathbf{1}, \mathbf{3}, -1/3)$ and $\rho = (\rho^+_1, \rho^0, \rho^+_2)^\mathrm{T} \sim (\mathbf{1}, \mathbf{3}, 2/3)$. The complete set of fields in the SUSY 3-3-1 RN is given in [54, 55].

In the model under consideration, the superpotential is given by

$$\begin{aligned} W_2 = {} & \mu_{0a}\hat{L}_a\hat{\eta}' + \mu_{1a}\hat{L}_a\hat{\chi}' + \mu_\eta\hat{\eta}\hat{\eta}' + \mu_\chi\hat{\chi}\hat{\chi}' + \mu_2\hat{\eta}\hat{\chi}' \\ & + \mu_3\hat{\chi}\hat{\eta}' + \mu_\rho\hat{\rho}\hat{\rho}' , \\ W_3 = {} & \lambda_{1ab}\hat{L}_a\hat{\rho}'\hat{l}^c_b + \lambda_{2a}\epsilon\hat{L}_a\hat{\chi}\hat{\rho} + \lambda_{3a}\epsilon\hat{L}_a\hat{\eta}\hat{\rho} + \lambda_{4ab}\epsilon\hat{L}_a\hat{L}_b\hat{\rho} \\ & + \kappa_{1i}\hat{Q}_3\hat{\eta}'\hat{u}^c_i + \kappa'_1\hat{Q}_3\hat{\eta}'\hat{u}'^c + \kappa_{2i}\hat{Q}_3\hat{\chi}'\hat{u}^c_i \\ & + \kappa'_2\hat{Q}_3\hat{\chi}'\hat{u}'^c + \kappa_{3\alpha i}\hat{Q}_\alpha\hat{\eta}\hat{d}^c_i + \kappa'_{3\alpha\beta}\hat{Q}_\alpha\hat{\eta}\hat{d}'^c_\beta \\ & + \kappa_{4\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}^c_i + \kappa'_{4\alpha}\hat{Q}_\alpha\hat{\rho}\hat{u}'^c + \kappa_{5i}\hat{Q}_3\hat{\rho}'\hat{d}^c_i + \kappa'_{5\beta}\hat{Q}_3\hat{\rho}'\hat{d}^c_\beta \\ & + \kappa_{6\alpha i}\hat{Q}_\alpha\hat{\chi}\hat{d}^c_i + \kappa'_{6\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{d}'^c_\beta + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f'_1\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}' \\ & + \zeta_{\alpha\beta\gamma}\epsilon\hat{Q}_\alpha\hat{Q}_\beta\hat{Q}_\gamma + \lambda'_{\alpha ai}\hat{Q}_\alpha\hat{L}_a\hat{d}^c_i + \lambda''_{ijk}\hat{d}^c_i\hat{u}^c_j\hat{d}^c_k \\ & + \xi_{1ij\beta}\hat{d}^c_i\hat{u}^c_j\hat{d}'^c_\beta + \xi_{2\alpha a\beta}\hat{Q}_\alpha\hat{L}_a\hat{d}'^c_\beta + \xi_{3i\beta}\hat{d}^c_i\hat{u}'^c\hat{d}'^c_\beta \\ & + \xi_{4ij}\hat{d}^c_i\hat{u}'^c\hat{d}^c_j + \xi_{5\alpha i\beta}\hat{d}'^c_\alpha\hat{u}^c_i\hat{d}'^c_\beta + \xi_{6\alpha\beta}\hat{d}'^c_\alpha\hat{u}'^c\hat{d}'^c_\beta . \end{aligned} \tag{46}$$

Applying the conditions coming from (29) to (46), we get the following equations:

$$\begin{aligned} & n_{\mathrm{L}} + n_{\eta'} = 0 , \; n_{\mathrm{L}} + n_{\chi'} = 0 , \; n_\eta + n_{\eta'} = 0 , \\ & n_\chi + n_{\chi'} = 0 , \; n_\eta + n_{\chi'} = 0 , \; n_\chi + n_{\eta'} = 0 , \\ & n_\rho + n_{\rho'} = 0 , \; n_{\mathrm{L}} + n_{\rho'} + n_l = 0 , \\ & n_{\mathrm{L}} + n_\chi + n_\rho = 0 , \; n_{\mathrm{L}} + n_\eta + n_\rho = 0 , \; 2n_{\mathrm{L}} + n_\rho = 0 , \\ & n_{Q_3} + n_{\eta'} + n_u = 0 , \; n_{Q_3} + n_{\eta'} + n_{u'} = 0 , \\ & n_{Q_3} + n_{\chi'} + n_u = 0 , \; n_{Q_3} + n_{\chi'} + n_{u'} = 0 , \\ & n_{Q_3} + n_{\rho'} + n_d = 0 , \; n_{Q_3} + n_{\rho'} + n_{d'} = 0 , \\ & n_{Q_\alpha} + n_\eta + n_d = 0 , \; n_{Q_\alpha} + n_\eta + n_{d'} = 0 , \\ & n_{Q_\alpha} + n_\chi + n_d = 0 , \; n_{Q_\alpha} + n_\chi + n_{d'} = 0 , \\ & n_{Q_\alpha} + n_\rho + n_u = 0 , \; n_{Q_\alpha} + n_\rho + n_{u'} = 0 , \; 3n_{Q_\alpha} = 0 , \\ & n_\rho + n_\chi + n_\eta = 0 , \; n_{\rho'} + n_{\chi'} + n_{\eta'} = 0 , \\ & n_{Q_\alpha} + n_{\mathrm{L}} + n_d = 0 , \; n_{Q_\alpha} + n_{\mathrm{L}} + n_{d'} = 0 , \\ & 2n_d + n_u = 0 , \; n_d + n_{d'} + n_u = 0 , \\ & n_d + n_{d'} + n_{u'} = 0 , \; 2n_d + n_{u'} = 0 , \; 2n_{d'} + n_u = 0 , \\ & 2n_{d'} + n_{u'} = 0 . \end{aligned} \tag{47}$$

Choosing the following $R$-charges:

$$\begin{aligned} & n_\eta = n_{\eta'} = n_\chi = n_{\chi'} = n_\rho = n_{\rho'} = 0 , \\ & n_{\mathrm{L}} = n_{Q_\alpha} = n_{Q_3} = 1/2 , \\ & n_l = n_u = n_d = n_{u'} = n_{d'} = -1/2 , \end{aligned} \tag{48}$$

and looking at (27), it is easy to see that all the fields $\eta$, $\eta'$, $\chi$, $\chi'$, $\rho$, $\rho'$, $L$, $Q_\alpha$, $Q_3$, $l$, $u$, $u'$, $d$ and $d'$ have $R$-charge equal to 1, while their superpartners have opposite $R$-charge, similar to that in the MSSM. The terms which satisfy the defined above symmetry (48) are

$$\begin{aligned} W = {} & \frac{\mu_\eta}{2}\hat{\eta}\hat{\eta}' + \frac{\mu_\chi}{2}\hat{\chi}\hat{\chi}' + \frac{\mu_\rho}{2}\hat{\rho}\hat{\rho}' + \frac{\mu_2}{2}\hat{\eta}\hat{\chi}' + \frac{\mu_3}{2}\hat{\chi}\hat{\eta}' \\ & + \frac{1}{3}\Big[\lambda_{1ab}\hat{L}_a\hat{\rho}'\hat{l}^c_b + \kappa_{1i}\hat{Q}_3\hat{\eta}'\hat{u}^c_i + \kappa'_1\hat{Q}_3\hat{\eta}'\hat{u}'^c + \kappa_{2i}\hat{Q}_3\hat{\chi}'\hat{u}^c_i \\ & + \kappa'_2\hat{Q}_3\hat{\chi}'\hat{u}'^c + \kappa_{3\alpha i}\hat{Q}_\alpha\hat{\eta}\hat{d}^c_i + \kappa'_{3\alpha\beta}\hat{Q}_\alpha\hat{\eta}\hat{d}'^c_\beta + \kappa_{4\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}^c_i \\ & + \kappa'_{4\alpha}\hat{Q}_\alpha\hat{\rho}\hat{u}'^c + \kappa_{5i}\hat{Q}_3\hat{\rho}'\hat{d}^c_i + \kappa'_{5\beta}\hat{Q}_3\hat{\rho}'\hat{d}^c_\beta \\ & + \kappa_{6\alpha i}\hat{Q}_\alpha\hat{\chi}\hat{d}^c_i + \kappa'_{6\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{d}'^c_\beta \\ & + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f'_1\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}'\Big] . \end{aligned} \tag{49}$$

Because of the lepton content of the model considered, the lepton number $L$ obviously does not commute with the gauge symmetry. However, a new conserved charge $\mathcal{L}$ can be constructed through $L$ by making the linear combination $L = x\lambda_3 + y\lambda_8 + \mathcal{L}I$ where $\lambda_3$ and $\lambda_8$ are the diagonal generators of the $SU(3)_{\mathrm{L}}$ group. Applying this operator on a lepton triplet, the coefficients will be defined

$$L = \frac{2}{\sqrt{3}}\lambda_8 + \mathcal{L}I . \tag{50}$$

Moreover, it is useful to produce another conserved charge, $\mathcal{B}$, which itself is the usual baryon number, $B = \mathcal{B}I$. Thus, the $R$-parity in this model can be re-expressed via the spin $S$ and new charges $\mathcal{L}$ and $\mathcal{B}$:

$$R\text{-parity} = (-1)^{2S}(-1)^{3(\mathcal{B}+\mathcal{L})} , \tag{51}$$

where the charges $\mathcal{B}$ and $\mathcal{L}$ for the multiplets are defined as follows [52]

| *Triplet* | $L$ | $Q_3$ | $\chi$ | $\eta$ | $\rho$ |
|---|---|---|---|---|---|
| $\mathcal{B}$ *charge* | 0 | 1/3 | 0 | 0 | 0 |
| $\mathcal{L}$ *charge* | 1/3 | −2/3 | 4/3 | −2/3 | −2/3 |

(52)

| *Anti-Triplet* | $Q_\alpha$ | $\chi'$ | $\eta'$ | $\rho'$ |
|---|---|---|---|---|
| $\mathcal{B}$ *charge* | 1/3 | 0 | 0 | 0 |
| $\mathcal{L}$ *charge* | 2/3 | −4/3 | 2/3 | 2/3 |

(53)

| *Singlet* | $l^c$ | $u^c$ | $d^c$ | $u'^c$ | $d'^c$ |
|---|---|---|---|---|---|
| $\mathcal{B}$ *charge* | 0 | −1/3 | −1/3 | −1/3 | −1/3 |
| $\mathcal{L}$ *charge* | −1 | 0 | 0 | 2 | −2 |

(54)

From the superpotential given in (49), it is easy to see that the charged leptons gain mass only from the term

$$-\frac{\lambda_{1ab}}{3}L_a\rho' l^c_b + \mathrm{h.c.} \tag{55}$$

The Higgs fields can have VEVs given by

$$\begin{aligned} & \langle\rho\rangle = (0, u, 0)^{\mathrm{T}} , \; \langle\rho'\rangle = (0, u', 0)^{\mathrm{T}} , \\ & \langle\eta\rangle = (v, 0, 0)^{\mathrm{T}} , \; \langle\eta'\rangle = (v', 0, 0)^{\mathrm{T}} , \\ & \langle\chi\rangle = (0, 0, w)^{\mathrm{T}} , \; \langle\chi'\rangle = (0, 0, w')^{\mathrm{T}} . \end{aligned} \tag{56}$$

Combining (55) and (56) we get the mass terms

$$-\frac{\lambda_{1ab}}{3}(l_a l^c_b + \bar{l}_a \bar{l}^c_b)u' , \tag{57}$$

which lead to the following mass matrix:

$$X = \begin{pmatrix} \frac{\lambda_{111}}{3}u' & \frac{\lambda_{112}}{3}u' & \frac{\lambda_{113}}{3}u' \\ \frac{\lambda_{121}}{3}u' & \frac{\lambda_{122}}{3}u' & \frac{\lambda_{123}}{3}u' \\ \frac{\lambda_{131}}{3}u' & \frac{\lambda_{132}}{3}u' & \frac{\lambda_{133}}{3}u' \end{pmatrix} .$$

Hence, all the charged leptons get mass. Notice that only the VEV of $\rho'$ is enough to give the charged leptons masses.

Recall that due to conservation of the $R$-parity defined in (48), there are no terms which give neutrinos masses in the superpotential. Thus, in this case the neutrinos remain massless. On the other hand, the gaugino mass terms are given by

$$\mathcal{L}_{\mathrm{GMT}} = -\frac{m_\lambda}{2}\left[\sum_{a=1}^{8}\left(\lambda_A^a\lambda_A^a + \bar{\lambda}_A^a\bar{\lambda}_A^a\right)\right] - \frac{m'}{2}\left(\lambda_B\lambda_B + \bar{\lambda}_B\bar{\lambda}_B\right). \quad (58)$$

Recall that in this model the non-Hermitian gauge bosons are defined as

$$\begin{aligned} \sqrt{2}\, W_m^{\pm} &= V_m^1 \mp \mathrm{i}V_m^2 , \\ \sqrt{2}\, Y_m^{\pm} &= V_m^6 \pm \mathrm{i}V_m^7 , \\ \sqrt{2}\, X_m^{0} &= V_m^4 - \mathrm{i}V_m^5 . \end{aligned} \quad (59)$$

According to these equations, the gauginos of the model are defined as

$$\begin{aligned} \sqrt{2}\, \lambda_W^{\pm} &= \lambda_A^1 \mp \mathrm{i}\lambda_A^2 , \\ \sqrt{2}\, \lambda_Y^{\pm} &= \lambda_A^6 \pm \mathrm{i}\lambda_A^7 , \\ \sqrt{2}\, \lambda_{X^0} &= \lambda_A^4 - \mathrm{i}\lambda_A^5 . \end{aligned} \quad (60)$$

Then, in terms of these fields, (58) can be rewritten as

$$-m_\lambda\left(\lambda_W^-\lambda_W^+ + \lambda_Y^-\lambda_Y^+ + \lambda_{X^0}\lambda_{X^{0*}}\right) - \frac{m_\lambda}{2}\left(\lambda_A^3\lambda_A^3 + \lambda_A^8\lambda_A^8\right) - \frac{m'}{2}\lambda_B\lambda_B + \mathrm{h.c.} \quad (61)$$

The mass matrix of charginos and higgsinos arises from the following Lagrangian:

$$\begin{aligned} \mathcal{L}_{H\tilde{H}\tilde{V}} = &-\frac{\mathrm{i}g}{\sqrt{2}}\Big[\bar{\tilde{\eta}}\lambda^a\eta\bar{\lambda}_A^a - \bar{\eta}\lambda^a\tilde{\eta}\lambda_A^a + \bar{\tilde{\rho}}\lambda^a\rho\bar{\lambda}_A^a - \bar{\rho}\lambda^a\tilde{\rho}\lambda_A^a \\ &+ \bar{\tilde{\chi}}\lambda^a\chi\bar{\lambda}_A^a - \bar{\chi}\lambda^a\tilde{\chi}\lambda_A^a \\ &- \bar{\tilde{\eta}}'\lambda^{*a}\eta'\bar{\lambda}_A^a + \bar{\eta}'\lambda^{*a}\tilde{\eta}'\lambda_A^a - \bar{\tilde{\rho}}'\lambda^{*a}\rho'\bar{\lambda}_A^a \\ &+ \bar{\rho}'\lambda^{*a}\tilde{\rho}'\lambda_A^a - \bar{\tilde{\chi}}'\lambda^{*a}\chi'\bar{\lambda}_A^a + \bar{\chi}'\lambda^{*a}\tilde{\chi}'\lambda_A^a\Big] \\ &- \frac{\mathrm{i}g'}{\sqrt{2}}\Big[-\frac{1}{3}\left(\bar{\tilde{\eta}}\eta\bar{\lambda}_B - \bar{\eta}\tilde{\eta}\lambda_B\right) - \frac{1}{3}\left(\bar{\tilde{\chi}}\chi\bar{\lambda}_B - \bar{\chi}\tilde{\chi}\lambda_B\right) \\ &+ \frac{1}{3}\left(\bar{\tilde{\eta}}'\eta'\bar{\lambda}_B - \bar{\eta}'\tilde{\eta}'\lambda_B\right) \\ &+ \frac{1}{3}\left(\bar{\tilde{\chi}}'\chi'\bar{\lambda}_B - \bar{\chi}'\tilde{\chi}'\lambda_B\right) + \frac{2}{3}\left(\bar{\tilde{\rho}}\rho\bar{\lambda}_B - \bar{\rho}\tilde{\rho}\lambda_B\right) \\ &- \frac{2}{3}\left(\bar{\tilde{\rho}}'\rho'\bar{\lambda}_B - \bar{\rho}'\tilde{\rho}'\lambda_B\right)\Big]. \end{aligned} \quad (62)$$

Using (60), we can rewrite the Lagrangian as

$$\begin{aligned} \mathcal{L}_{H\tilde{H}\tilde{V}} = &+\mathrm{i}g\Big[v\tilde{\eta}^-\lambda_W^+ + w\tilde{\chi}^-\lambda_Y^+ + u\left(\tilde{\rho}_1^+\lambda_W^- + \tilde{\rho}_2^+\lambda_Y^-\right) \\ &- v'\tilde{\eta}'^+\lambda_W^- - w'\tilde{\chi}'^+\lambda_Y^- \\ &- u'\left(\tilde{\rho}_1'^-\lambda_W^+ + \tilde{\rho}_2'^-\lambda_Y^+\right)\Big] - \frac{\mathrm{i}gv}{\sqrt{2}}\tilde{\eta}_1^0\lambda_A^3 - \mathrm{i}gv\tilde{\eta}_2^0\lambda_{X^0} \\ &- \frac{\mathrm{i}gv}{\sqrt{2}}\tilde{\eta}_1^0\lambda_A^8 - \mathrm{i}gw\tilde{\chi}_1^0\lambda_{X^{0*}} + \mathrm{i}gw\sqrt{\frac{2}{3}}\tilde{\chi}_2^0\lambda_A^8 \\ &+ \frac{\mathrm{i}gu}{\sqrt{2}}\tilde{\rho}^0\lambda_A^3 - \frac{\mathrm{i}gu}{\sqrt{6}}\tilde{\rho}^0\lambda_A^8 + \frac{\mathrm{i}gv'}{\sqrt{2}}\tilde{\eta}_1'^0\lambda_A^3 + \mathrm{i}gv'\tilde{\eta}_2'^0\lambda_{X^{0*}} \\ &+ \frac{\mathrm{i}gv'}{\sqrt{6}}\tilde{\eta}_1'^0\lambda_A^8 + \mathrm{i}gw'\tilde{\chi}_1'^0\lambda_{X^0} - \mathrm{i}gw'\sqrt{\frac{2}{3}}\tilde{\chi}_2'^0\lambda_A^8 \\ &+ \frac{\mathrm{i}gu'}{\sqrt{6}}\tilde{\rho}'^0\lambda_A^8 - \frac{\mathrm{i}gu'}{\sqrt{2}}\tilde{\rho}'^0\lambda_A^3 - \mathrm{i}g'u\sqrt{\frac{2}{3}}\tilde{\rho}^0\lambda_B \\ &+ \frac{\mathrm{i}g'v}{3\sqrt{2}}\tilde{\eta}_1^0\lambda_B + \frac{\mathrm{i}g'w}{3\sqrt{2}}\tilde{\chi}_2^0\lambda_B - \frac{\mathrm{i}g'v'}{3\sqrt{2}}\tilde{\eta}_1'^0\lambda_B \\ &- \frac{\mathrm{i}g'w'}{3\sqrt{2}}\tilde{\chi}_2'^0\lambda_B + \mathrm{i}g'u'\sqrt{\frac{2}{3}}\tilde{\rho}^0\lambda_B + \mathrm{h.c.} \end{aligned} \quad (63)$$

The mass terms of the higgsinos are given by

$$\begin{aligned} &-\frac{\mu_\eta}{2}\tilde{\eta}\tilde{\eta}' - \frac{\mu_\chi}{2}\tilde{\chi}\tilde{\chi}' - \frac{\mu_\rho}{2}\tilde{\rho}\tilde{\rho}' - \frac{f_1}{3}\epsilon\left(\tilde{\rho}\tilde{\chi}\eta + \rho\tilde{\chi}\tilde{\eta} + \tilde{\rho}\chi\tilde{\eta}\right) \\ &-\frac{f_1'}{3}\epsilon\left(\tilde{\rho}'\tilde{\chi}'\eta' + \rho'\tilde{\chi}'\tilde{\eta}' + \tilde{\rho}'\chi'\tilde{\eta}'\right) + \mathrm{h.c.} \end{aligned}$$

In the terms of field components, the above expression can be rewritten as

$$\begin{aligned} &-\frac{\mu_\eta}{2}\left(\tilde{\eta}_1^0\tilde{\eta}_1'^0 + \tilde{\eta}^-\tilde{\eta}'^+ + \tilde{\eta}_2^0\tilde{\eta}_2'^0\right) \\ &-\frac{\mu_\rho}{2}\left(\tilde{\rho}_1^+\tilde{\rho}_1'^- + \tilde{\rho}^0\tilde{\rho}'^0 + \tilde{\rho}_2^+\tilde{\rho}_2'^-\right) \\ &-\frac{\mu_\chi}{2}\left(\tilde{\chi}_1^0\tilde{\chi}_1'^0 + \tilde{\chi}^-\tilde{\chi}'^+ + \tilde{\chi}_2^0\tilde{\chi}_2'^0\right) - \frac{f_1}{3}\Big[\left(\tilde{\rho}^0\tilde{\chi}_2^0 - \tilde{\rho}_2^+\tilde{\chi}^-\right)\eta_1^0 \\ &+\left(\tilde{\rho}_1^+\tilde{\chi}^- - \tilde{\rho}^0\tilde{\chi}_1^0\right)\eta_2^0 + \left(\tilde{\chi}_2^0\tilde{\eta}_1^0 - \tilde{\chi}_1^0\tilde{\eta}_2^0\right)\rho^0 \\ &+\left(\tilde{\rho}_2^+\tilde{\eta}^- - \tilde{\rho}^0\tilde{\eta}_2^0\right)\chi_1^0 + \left(\tilde{\rho}^0\tilde{\eta}_1^0 - \tilde{\rho}_1^+\tilde{\eta}^-\right)\chi_2^0\Big] \\ &-\frac{f_1'}{3}\Big[\left(\tilde{\rho}'^0\tilde{\chi}_2'^0 - \tilde{\rho}_2'^-\tilde{\chi}'^+\right)\eta_1'^0 \\ &+\left(\tilde{\rho}_1'^-\tilde{\chi}'^+ - \tilde{\rho}'^0\tilde{\chi}_1'^0\right)\eta_2'^0 + \left(\tilde{\chi}_2'^0\tilde{\eta}_1'^0 - \tilde{\chi}_1'^0\tilde{\eta}_2'^0\right)\rho'^0 \\ &+\left(\tilde{\rho}_2'^-\tilde{\eta}'^+ - \tilde{\rho}'^0\tilde{\eta}_2'^0\right)\chi_1'^0 + \left(\tilde{\rho}'^0\tilde{\eta}_1'^0 - \tilde{\rho}_1'^-\tilde{\eta}'^+\right)\chi_2'^0\Big] + \mathrm{h.c.} \end{aligned} \quad (64)$$

The neutral gauginos and higgsinos are mixed by the matrix $N$:

$$\tilde{\chi}_\alpha^0 = \sum_{\substack{\beta=\text{gauginos,}\\ \text{higgsinos}}} N_{\alpha\beta}\psi_\beta^0 , \quad (65)$$

where the $\tilde{\chi}_\alpha^0$ form fifteen physical Majorana particles. The charged gauginos mix with higgsinos via two mixing matrices $C_{\alpha\beta}^{\pm}$:

$$\tilde{\chi}_\alpha^{\pm} = \sum_{\substack{\beta=\text{gauginos,}\\ \text{higgsinos}}} C_{\alpha\beta}^{\pm}\psi_\beta^{\pm} , \quad (66)$$

to form six physical Dirac particles. These matrices will be presented in the next section.

### 3.1 The discrete symmetry for proton stability and neutrino masses in SUSY 3-3-1 RN

As before, if we choose the $R$-charges as follows:

$$\begin{aligned} n_{\mathrm{L}} &= n_\eta = n_\chi = \frac{1}{2}\,, \\ n_{\eta'} &= n_{\chi'} = n_u = n_{u'} = -\frac{1}{2}\,, \\ n_{Q_3} &= n_{\rho'} = 1\,,\ n_d = n_{d'} = -2\,, \\ n_\rho &= -1\,,\ n_{Q_\alpha} = \frac{3}{2}\,,\ n_l = -\frac{3}{2}\,, \end{aligned} \tag{67}$$

then the terms under this symmetry are obtained by

$$\begin{aligned} W = {} & \frac{1}{2}\Big(\mu_{0a}\hat{L}_a\hat{\eta}' + \mu_{1a}\hat{L}_a\hat{\chi}' + \mu_\eta\hat{\eta}\hat{\eta}' + \mu_\chi\hat{\chi}\hat{\chi}' \\ & + \mu_2\hat{\eta}\hat{\chi}' + \mu_3\hat{\chi}\hat{\eta}' + \mu_\rho\hat{\rho}\hat{\rho}'\Big) \\ & + \frac{1}{3}\Big(\lambda_{1ab}\hat{L}_a\hat{\rho}'\hat{l}^c_b + \lambda_{2a}\epsilon\hat{L}_a\hat{\chi}\hat{\rho} + \lambda_{3a}\epsilon\hat{L}_a\hat{\eta}\hat{\rho} \\ & + \lambda_{4ab}\epsilon\hat{L}_a\hat{L}_b\hat{\rho} + \kappa_{1i}\hat{Q}_3\hat{\eta}'\hat{u}^c_i + \kappa'_1\hat{Q}_3\hat{\eta}'\hat{u}'^c \\ & + \kappa_{2i}\hat{Q}_3\hat{\chi}'\hat{u}^c_i + \kappa'_2\hat{Q}_3\hat{\chi}'\hat{u}'^c + \kappa_{3\alpha i}\hat{Q}_\alpha\hat{\eta}\hat{d}^c_i \\ & + \kappa'_{3\alpha\beta}\hat{Q}_\alpha\hat{\eta}\hat{d}'^c_\beta + \kappa_{4\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}^c_i + \kappa'_{4\alpha}\hat{Q}_\alpha\hat{\rho}\hat{u}'^c \\ & + \kappa_{5i}\hat{Q}_3\hat{\rho}'\hat{d}^c_i + \kappa'_{5\beta}\hat{Q}_3\hat{\rho}'\hat{d}^c_\beta + \kappa_{6\alpha i}\hat{Q}_\alpha\hat{\chi}\hat{d}^c_i \\ & + \kappa'_{6\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{d}'^c_\beta + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f'_1\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}' \\ & + \lambda'_{\alpha a i}\hat{Q}_\alpha\hat{L}_a\hat{d}^c_i + \xi_{2\alpha a\beta}\hat{Q}_\alpha\hat{L}_a\hat{d}'^c_\beta\Big). \end{aligned} \tag{68}$$

In this case, it is easy to see that the fields $L,\ l,\ Q_\alpha,\ u,\ u'$, $\tilde{\eta},\ \tilde{\eta}',\ \tilde{\chi},\ \tilde{\chi}',\ \rho,\ \rho',\ \tilde{Q}_3,\ \tilde{d}$ and $\tilde{d}'$ have $R$-charge equal to 1, while the others fields have opposite $R$-charge.

The superpotential in (68) provides us the mass terms for leptons and higgsinos:

$$\begin{aligned} & -\frac{\mu_{0a}}{2}L_a\tilde{\eta}' - \frac{\mu_{1a}}{2}L_a\tilde{\chi}' - \frac{\mu_2}{2}\tilde{\eta}\tilde{\chi}' - \frac{\mu_3}{2}\tilde{\chi}\tilde{\eta}' \\ & -\frac{\lambda_{2a}}{3}\left(L_a\tilde{\chi}\rho + \tilde{\rho}L_a\chi\right) - \frac{\lambda_{3a}}{3}\left(L_a\tilde{\eta}\rho + \tilde{\rho}L_a\eta\right) \\ & -\frac{\lambda_{4ab}}{3}L_aL_b\rho + \mathrm{h.c.}\,, \end{aligned} \tag{69}$$

which in terms of field components get the form

$$\begin{aligned} & -\frac{\mu_{0a}}{2}\left(\nu_a\tilde{\eta}'^0_1 + l_a\tilde{\eta}'^+ + \nu^c_a\tilde{\eta}'^0_2\right) \\ & -\frac{\mu_{1a}}{2}\left(\nu_a\tilde{\chi}'^0_1 + l_a\tilde{\chi}'^+ + \nu^c_a\tilde{\chi}'^0_2\right) \\ & -\frac{\mu_2}{2}\left(\tilde{\eta}^0_1\tilde{\chi}'^0_1 + \tilde{\eta}^-\tilde{\chi}'^+ + \tilde{\eta}^0_2\tilde{\chi}'^0_2\right) \\ & -\frac{\mu_3}{2}\left(\tilde{\chi}^0_1\tilde{\eta}'^0_1 + \tilde{\chi}^-\tilde{\eta}'^+ + \tilde{\chi}^0_2\tilde{\eta}'^0_2\right) \\ & -\frac{\lambda_{2a}}{3}\Big[\left(\nu^c_a\tilde{\chi}^0_1 - \nu_a\tilde{\chi}^0_2\right)\rho^0 \\ & + \left(\tilde{\rho}^0\nu^c_a - \tilde{\rho}^+_2 l_a\right)\chi^0_1 + \left(\tilde{\rho}^+_1 l_a - \tilde{\rho}^0\nu_a\right)\chi^0_2\Big] \\ & -\frac{\lambda_{3a}}{3}\Big[\left(\nu^c_a\tilde{\eta}^0_1 - \nu_a\tilde{\eta}^0_2\right)\rho^0 + \left(\tilde{\rho}^0\nu^c_a - \tilde{\rho}^+_2 l_a\right)\eta^0_1 \\ & + \left(\tilde{\rho}^+_1 l_a - \tilde{\rho}^0\nu_a\right)\eta^0_2\Big] - \frac{\lambda_{4ab}}{3}\left(\nu^c_a\nu_b - \nu^c_b\nu_a\right)\rho^0 + \mathrm{h.c.} \end{aligned} \tag{70}$$

It is easy to see that the above Lagrangian gives mass terms for neutrinos and forbids proton decay.

## 4 Fermion masses

With the above mass terms, we get the mass matrices for the neutral and charged fermions, respectively. Diagonalizing these matrices, we get the physical masses for the fermions.

### 4.1 Masses of the neutral fermions

Mass Lagrangians for the neutral fermions are easily obtained by

$$\begin{aligned} \mathcal{M}_{\mathrm{neutral}} = {} & -\frac{1}{2}\Big[\mu_{0a}\left(\nu_{a\mathrm{L}}\tilde{\eta}'^0_1 + \nu^c_{a\mathrm{L}}\tilde{\eta}'^0_2\right) \\ & + \mu_{1a}\left(\nu_{a\mathrm{L}}\tilde{\chi}'^0_1 + \nu^c_{a\mathrm{L}}\tilde{\chi}'^0_2\right) + \mu_\eta\left(\tilde{\eta}^0_1\tilde{\eta}'^0_1 + \tilde{\eta}^0_2\tilde{\eta}'^0_2\right) \\ & + \mu_\chi\left(\tilde{\chi}^0_1\tilde{\chi}'^0_1 + \tilde{\chi}^0_2\tilde{\chi}'^0_2\right) + \mu_\rho\tilde{\rho}^0\tilde{\rho}'^0 \\ & + \mu_2\left(\tilde{\eta}^0_1\tilde{\chi}'^0_1 + \tilde{\eta}^0_2\tilde{\chi}'^0_2\right) + \mu_3\left(\tilde{\chi}^0_1\tilde{\eta}'^0_1 + \tilde{\chi}^0_2\tilde{\eta}'^0_2\right)\Big] \\ & -\frac{1}{3}\Big[\lambda_{2a}u\left(\nu^c_{a\mathrm{L}}\tilde{\chi}^0_1 - \nu_{a\mathrm{L}}\tilde{\chi}^0_2\right) - \lambda_{2a}w\nu_{a\mathrm{L}}\tilde{\rho}^0 \\ & + \lambda_{3a}u\left(\nu^c_{a\mathrm{L}}\tilde{\eta}^0_1 - \nu_{a\mathrm{L}}\tilde{\eta}^0_2\right) \\ & + \lambda_{3a}v\nu^c_{a\mathrm{L}}\tilde{\rho}^0 + \lambda_{4ab}u\left(\nu^c_{a\mathrm{L}}\nu_{b\mathrm{L}} - \nu^c_{b\mathrm{L}}\nu_{a\mathrm{L}}\right) \\ & + f_1v\tilde{\rho}^0\tilde{\chi}^0_2 + f_1w\tilde{\rho}^0\tilde{\eta}^0_1 \\ & + f_1u\left(\tilde{\chi}^0_2\tilde{\eta}^0_1 - \tilde{\chi}^0_1\tilde{\eta}^0_2\right) + f'_1v'\tilde{\rho}'^0\tilde{\chi}'^0_2 + f'_1w'\tilde{\rho}'^0\tilde{\eta}'^0_1 \\ & + f'_1u'\left(\tilde{\chi}'^0_2\tilde{\eta}'^0_1 - \tilde{\chi}'^0_1\tilde{\eta}'^0_2\right)\Big] \\ & -\frac{m_\lambda}{2}\left(2\lambda_{X^0}\lambda_{X^{0*}} + \lambda^3_A\lambda^3_A + \lambda^8_A\lambda^8_A\right) - \frac{m'}{2}\lambda_B\lambda_B \\ & + \frac{\mathrm{i}gv}{\sqrt{2}}\tilde{\eta}^0_1\lambda^3_A + \mathrm{i}gv\tilde{\eta}^0_2\lambda_{X^0} + \frac{\mathrm{i}gv}{\sqrt{6}}\tilde{\eta}^0_1\lambda^8_A \\ & + \mathrm{i}gw\tilde{\chi}^0_1\lambda_{X^{0*}} - \mathrm{i}gw\sqrt{\frac{2}{3}}\tilde{\chi}^0_2\lambda^8_A - \frac{\mathrm{i}gu}{\sqrt{2}}\tilde{\rho}^0\lambda^3_A \\ & + \frac{\mathrm{i}gu}{\sqrt{6}}\tilde{\rho}^0\lambda^8_A - \frac{\mathrm{i}gv'}{\sqrt{2}}\tilde{\eta}'^0_1\lambda^3_A - \mathrm{i}gv'\tilde{\eta}'^0_2\lambda_{X^{0*}} \\ & - \frac{\mathrm{i}gv'}{\sqrt{6}}\tilde{\eta}'^0_1\lambda^8_A - \mathrm{i}gw'\tilde{\chi}'^0_1\lambda_{X^0} + \mathrm{i}gw'\sqrt{\frac{2}{3}}\tilde{\chi}'^0_2\lambda^8_A \\ & - \frac{\mathrm{i}gu'}{\sqrt{6}}\tilde{\rho}'^0\lambda^8_A - \frac{\mathrm{i}gu'}{\sqrt{2}}\tilde{\rho}'^0\lambda^3_A \\ & + \mathrm{i}g'u\sqrt{\frac{2}{3}}\tilde{\rho}^0\lambda_B - \frac{\mathrm{i}g'v}{3\sqrt{2}}\tilde{\eta}^0_1\lambda_B - \frac{\mathrm{i}g'w}{3\sqrt{2}}\tilde{\chi}^0_2\lambda_B \\ & + \frac{\mathrm{i}g'v'}{3\sqrt{2}}\tilde{\eta}'^0_1\lambda_B + \frac{\mathrm{i}g'w'}{3\sqrt{2}}\tilde{\chi}'^0_2\lambda_B \\ & - \mathrm{i}g'u'\sqrt{\frac{2}{3}}\tilde{\rho}'^0\lambda_B + \mathrm{h.c.} \end{aligned}$$

In the basis $\Psi^0$ of the form

$$\begin{aligned} & \big(\nu_1\ \nu_2\ \nu_3\ \nu^c_1\ \nu^c_2\ \nu^c_3 - \mathrm{i}\lambda^3_A - \mathrm{i}\lambda_{X^0} - \mathrm{i}\lambda_{X^{0*}} - \mathrm{i}\lambda^8_A \\ & - \mathrm{i}\lambda_B\ \tilde{\eta}^0_1\ \tilde{\eta}'^0_1\ \tilde{\eta}^0_2\ \tilde{\eta}'^0_2\ \tilde{\chi}^0_1\ \tilde{\chi}'^0_1\ \tilde{\chi}^0_2\ \tilde{\chi}'^0_2\ \tilde{\rho}^0\ \tilde{\rho}'^0\big)\,, \end{aligned}$$

the mass Lagrangian can be written as follows:

$$-\frac{1}{2}(\Psi^0)^T Y^0 \Psi^0 + \text{h.c.}. \quad (71)$$

Here $Y^0$ is a symmetric matrix with the nonzero elements given in Appendix , where the mass eigenstates are given by

$$\tilde{\chi}_i^0 = N_{ij}\Psi_j^0, \; j = 1, \cdots, 21. \quad (72)$$

The mass matrix of the neutral fermions consists of three parts. (a) The first part $M_\nu$ is the $6\times 6$ mass matrix of the neutrinos which belongs to the 3-3-1 model with right-handed neutrinos; (b) the second part $M_N$ is the $15\times 15$ mass matrix of the neutralinos which exists only in its supersymmetric version; (c) the last part $M_{\nu N}$ arises due to mixing among the neutrinos and neutralinos.

Thus, the mass matrix for the neutral fermions is

$$Y^0 = \begin{pmatrix} M_\nu & M_{\nu N} \\ M_{\nu N}^{\mathrm{T}} & M_N \end{pmatrix}. \quad (73)$$

To keep consistence with the low-energy effective theories, some Yukawa couplings in the sub-matrices will be fixed in terms of fine-tunings as needed in this model. First, we know that the mass matrix $M_\nu$ gives three Dirac eigenstates. Two of them have degenerate eigenvalues, $m_\nu = \frac{2u}{3}\sqrt{\lambda_{412}^2 + \lambda_{413}^2 + \lambda_{423}^2}$, and the other one is massless. Thus, it is easy to identify the mass splitting $m_\nu$ as the value of measured atmospheric neutrino mass difference $\Delta m_{\text{atm}} \sim 5\times 10^{-2}$ eV. By putting $\lambda_{412} = \lambda_{413} = \lambda_{423}$, we get

$$\lambda_{412} = \lambda_{413} = \lambda_{423} = 2\times 10^{-13}, \\ \lambda_{421} = \lambda_{431} = \lambda_{432} = -2\times 10^{-13}. \quad (74)$$

It is worth emphasizing that when the mixing terms are turned on, this inverted spectrum will not only give rise to mass splitting between the two degenerate Dirac states, but it will also split each Dirac pair into two non-degenerate Majorana states, resulting in a spectrum with six Majorana eigenstates with four heavier ones and two light ones. Here we are assuming that the solar oscillation is between the two heavier Majorana states.

Finally, to keep to the mass constraints from astrophysics and cosmology [75] as well as to be consistent with all the earlier analyses [57], the parameters in the mass matrix $M_N$ can be chosen as in this typical example by

$$\mu_\eta = 300\,\text{GeV}, \; \mu_\rho = 500\,\text{GeV}, \; \mu_\chi = 700\,\text{GeV}, \\ \mu_2 = 50\,\text{GeV}, \; \mu_3 = 200\,\text{GeV}, \; m' = 2000\,\text{GeV}, \\ \mu_\lambda = 3000\,\text{GeV}, \; f_1 = 1.8, \; f_1' = 10^{-3}. \quad (75)$$

Here, in this model, the Higgs bosons' VEVs are fixed as follows:

$$v_\eta = 15\,\text{GeV}, \; v_{\eta'} = 10\,\text{GeV}, \\ v_\rho = 244.9\,\text{GeV}, \; v_{\rho'} = 13\,\text{GeV}, \\ v_{\chi_2} = v_{\chi_2'} = 1000\,\text{GeV}, \quad (76)$$

and by the value of $g$ given in [75].

Now, mixing terms arise to correct the effective ones; thus, in some ways we can get the physical masses. In the first case, the parameters in the mixing matrix $M_{\nu N}$ can be chosen as follows.

1. For the dimensionless parameters:

$$\lambda_{21} = 0, \; \lambda_{22} = 0, \; \lambda_{23} = 0, \\ \lambda_{31} = 0, \; \lambda_{32} = 0, \; \lambda_{33} = 0. \quad (77)$$

2. For the mass-scale parameters (in GeV):

$$\mu_{01} = 10^{-4}, \; \mu_{02} = 10^{-4}, \; \mu_{03} = 0, \\ \mu_{11} = 10^{-5}, \; \mu_{12} = 0, \; \mu_{13} = 0. \quad (78)$$

Hence, the neutralinos obtain masses in GeV as follows:

$$3210.2199, \; 3152.6352, \; 3152.6352, \; 3004.7008, \; 2087.1869, \\ 693.1140, \; 672.9818, \; 367.2895, \; 281.0245, \; 269.3481, \\ 151.7631, \; 115.9158, \; 48.4871, \; 46.7180, \; 39.5593. \quad (79)$$

The tauon neutrino gains masses in eV by

$$5.39600\times 10^{-5}, \; 4.12596\times 10^{-7}, \quad (80)$$

while the masses of the electron and muon neutrinos are (in eV)

$$6.31927\times 10^{-2}, \; 5.49914\times 10^{-2}, \\ 5.46530\times 10^{-2}, \; 4.70848\times 10^{-2}. \quad (81)$$

In the second case, the values of the parameters in the mixing matrix $M_{\nu N}$ are given in another way.

1. For the mass-scale parameters:

$$\mu_{01} = 0, \; \mu_{02} = 0, \; \mu_{03} = 0, \\ \mu_{11} = 0, \; \mu_{12} = 0, \; \mu_{13} = 0. \quad (82)$$

2. For the dimensionless parameters:

$$\lambda_{21} = 10^{-6}, \; \lambda_{22} = 10^{-6}, \; \lambda_{23} = 10^{-6}, \\ \lambda_{31} = 10^{-6}, \; \lambda_{32} = 0, \; \lambda_{33} = 0. \quad (83)$$

The masses (in GeV) of the neutralinos are

$$3210.2199, \; 3152.6352, \; 3152.6352, \; 3004.7008, \; 2087.1869, \\ 693.1140, \; 672.9818, \; 367.2895, \; 281.0245, \; 269.3481, \\ 151.7631, \; 115.9158, \; 48.4871, \; 46.7180, \; 39.5593. \quad (84)$$

The tauon neutrino in this case gains masses in eV by

$$1.95303\times 10^{-4}, \; 4.87078\times 10^{-5}. \quad (85)$$

Also, the electron and muon neutrinos have masses in eV

$$8.37329\times 10^{-2}, \; 5.52047\times 10^{-2}, \\ 5.51605\times 10^{-2}, \; 3.69629\times 10^{-2}. \quad (86)$$

Notice that the coupling constant $g'$, $\lambda_{4ab}$ and the parameter $m'$ appear only in the mass matrix of the neutral fermions, while those of the charged sector are $\lambda_{1ab}$.

As above, we give two typical examples of the values of the mixing terms which not only give the consistent mass spectra of the neutrinos but also keep a large enough hierarchy, so that the neutralinos gain masses satisfying the lower mass limit ($> 32.5$ GeV) from astrophysics and cosmology. Consequently, the neutrinos in this model yield inverted hierarchy mass patterns as shown in Fig. 1.

### 4.2 Masses of the charged fermions

The terms contributing to the masses of the charged fermions are

$$\begin{aligned}
\mathcal{M}_{\text{charged}} = & -\frac{\lambda_{1ab}}{3} l_a l_b^c u' - \frac{f_1}{3}\left(\tilde{\rho}_2^+ \tilde{\chi}^- v + \tilde{\rho}_1^+ \tilde{\eta}^- w\right) \\
& - \frac{f_1'}{3}\left(\tilde{\rho}_2'^- \tilde{\chi}'^+ v' + \tilde{\rho}_1'^- \tilde{\eta}'^+ w'\right) \\
& - m_\lambda \left(\lambda_W^- \lambda_W^+ + \lambda_Y^- \lambda_Y^+\right) + \mathrm{i}g\big[v \tilde{\eta}^- \lambda_W^+ + w \tilde{\chi}^- \lambda_Y^+ \\
& + u\left(\tilde{\rho}_1^+ \lambda_W^- + \tilde{\rho}_2^+ \lambda_Y^-\right) - v' \tilde{\eta}'^+ \lambda_W^- \\
& - w' \tilde{\chi}'^+ \lambda_Y^- - u'\left(\tilde{\rho}_1'^- \lambda_W^+ + \tilde{\rho}_2'^- \lambda_Y^+\right)\big] \\
& - \frac{\mu_\eta}{2} \tilde{\eta}^- \tilde{\eta}'^+ - \frac{\mu_\rho}{2}\left(\tilde{\rho}_1^+ \tilde{\rho}_1'^- + \tilde{\rho}_2^+ \tilde{\rho}_2'^-\right) - \frac{\mu_\chi}{2} \tilde{\chi}^- \tilde{\chi}'^+ \\
& - \frac{\mu_{0a}}{2} l_a \tilde{\eta}'^+ - \frac{\mu_{1a}}{2} l_a \tilde{\chi}'^+ - \frac{\mu_2}{2} \tilde{\eta}^- \tilde{\chi}'^+ - \frac{\mu_3}{2} \tilde{\chi}^- \tilde{\eta}'^+ \\
& - \frac{\lambda_{2a}}{3} l_a \tilde{\rho}_1^+ w + \frac{\lambda_{3a}}{3} l_a \tilde{\rho}_2^+ v + \text{h.c.}
\end{aligned}$$

To write the mass matrix, we will choose the following bases:

$$\begin{aligned}
\psi^- &= \left(l_1 l_2 l_3 - \mathrm{i}\lambda_W^- - \mathrm{i}\lambda_Y^- \tilde{\eta}^- \tilde{\chi}^- \tilde{\rho}_1'^- \tilde{\rho}_2'^-\right)^{\mathrm{T}} \\
\psi^+ &= \left(l_1^c l_2^c l_3^c \mathrm{i}\lambda_W^+ i\lambda_Y^+ \tilde{\eta}'^+ \tilde{\chi}'^+ \tilde{\rho}_1^+ \tilde{\rho}_2^+\right)^T
\end{aligned} \tag{87}$$

and define

$$\Psi^\pm = \left(\psi^+ \psi^-\right)^T . \tag{88}$$

With these definitions, the mass term is written in the form

$$-\frac{1}{2}(\Psi^\pm)^T Y^\pm \Psi^\pm + \text{h.c.}, \tag{89}$$

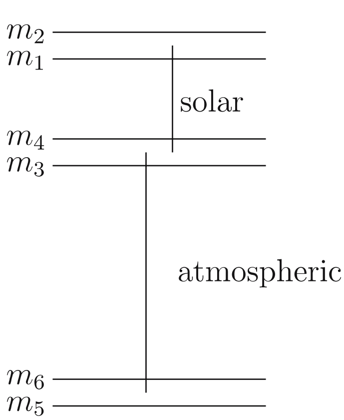


**Fig. 1.** The inverted hierarchy mass pattern of neutrinos in the model

where

$$Y^\pm = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} . \tag{90}$$

Then, the $X$ matrix is given by

$$X = \begin{pmatrix}
\frac{\lambda_{111}}{3}u' & \frac{\lambda_{112}}{3}u' & \frac{\lambda_{113}}{3}u' & 0 & 0 & \frac{1}{2}\mu_{01} & \frac{1}{2}\mu_{11} & \frac{\lambda_{21}}{3}w & -\frac{\lambda_{31}}{3}v \\
\frac{\lambda_{121}}{3}u' & \frac{\lambda_{122}}{3}u' & \frac{\lambda_{123}}{3}u' & 0 & 0 & \frac{1}{2}\mu_{02} & \frac{1}{2}\mu_{12} & \frac{\lambda_{22}}{3}w & -\frac{\lambda_{32}}{3}v \\
\frac{\lambda_{131}}{3}u' & \frac{\lambda_{132}}{3}u' & \frac{\lambda_{133}}{3}u' & 0 & 0 & \frac{1}{2}\mu_{03} & \frac{1}{2}\mu_{13} & \frac{\lambda_{23}}{3}w & -\frac{\lambda_{33}}{3}v \\
0 & 0 & 0 & -m_\lambda & 0 & -gv' & 0 & gu & 0 \\
0 & 0 & 0 & 0 & -m_\lambda & 0 & -gw' & 0 & gu \\
0 & 0 & 0 & gv & 0 & \frac{\mu_\eta}{2} & \frac{\mu_2}{2} & \frac{f_1}{3}w & 0 \\
0 & 0 & 0 & 0 & gw & \frac{\mu_3}{2} & \frac{\mu_\chi}{2} & 0 & \frac{f_1}{3}v \\
0 & 0 & 0 & -gu' & 0 & \frac{f_1'}{3}w' & 0 & \frac{\mu_\rho}{2} & 0 \\
0 & 0 & 0 & 0 & -gu' & 0 & \frac{f_1'}{3}v' & 0 & \frac{\mu_\rho}{2}
\end{pmatrix} .$$

The chargino mass matrix $Y^\pm$ is diagonalized by using two unitary matrices, $D$ and $E$, defined by

$$\tilde{\chi}_i^+ = D_{ij}\Psi_j^+, \ \ \tilde{\chi}_i^- = E_{ij}\Psi_j^-, \ \ i,j = 1,\cdots,9 . \tag{91}$$

The characteristic equation for the matrix $Y^\pm$ is

$$\det(Y^\pm - \lambda I) = \det\left[\begin{pmatrix} -\lambda & X^T \\ X & -\lambda \end{pmatrix}\right] = \det(\lambda^2 - X^T \cdot X) . \tag{92}$$

Since $X^T \cdot X$ is a symmetric matrix, $\lambda^2$ must be real and positive because $Y^\pm$ is also symmetric. Hence, to obtain eigenvalues, one only has to calculate $X^T \cdot X$. Then we can write the diagonal mass matrix as

$$M_{\text{SCM}} = E^* X D^{-1} . \tag{93}$$

To determine $E$ and $D$, it is useful to make the following observation:

$$M_{\text{SCM}}^2 = D X^{\mathrm{T}} \cdot X D^{-1} = E^* X \cdot X^{\mathrm{T}} (E^*)^{-1}, \tag{94}$$

which means that $D$ diagonalizes $X^T \cdot X$, while $E^*$ diagonalizes $X \cdot X^T$. In this case we can define the following Dirac spinors:

$$\Psi(\tilde{\chi}_i^+) = \left(\tilde{\chi}_i^+ \bar{\tilde{\chi}}_i^-\right)^{\mathrm{T}}, \ \ \Psi^c(\tilde{\chi}_i^-) = \left(\tilde{\chi}_i^- \bar{\tilde{\chi}}_i^+\right)^{\mathrm{T}}, \tag{95}$$

where $\tilde{\chi}_i^+$ is the particle and $\tilde{\chi}_i^-$ is the antiparticle [64, 76].

Now, to get mass values, all the parameters in the neutral sector should be kept in this sector of the charged fermions. Corresponding to the first case in the neutral sector, we have obtained the following masses (in GeV) for the charginos:

$$\begin{aligned}
& 3156.1474\,,\ 3004.4371\,,\ 665.1171\,,\ 263.3014\,, \\
& 209.0726\,, 45.9104\,.
\end{aligned} \tag{96}$$

The ordinary leptons gain masses in GeV as follows: $m_\tau = 1.7766$, $m_\mu = 0.1057$, $m_e = 0.00051$. In this case, the masses have been obtained by using the remaining set of the dimensionless parameters:

$$\begin{aligned}
&\lambda_{111} = 1.18\times 10^{-4}\,, \quad \lambda_{112} = 10^{-7}\,, \quad \lambda_{113} = 10^{-7}\,,\\
&\lambda_{121} = 10^{-7}, \quad \lambda_{122} = 2.44\times 10^{-2}\,, \quad \lambda_{123} = 10^{-7}\,,\\
&\lambda_{131} = 10^{-7}, \quad \lambda_{132} = 10^{-7}, \quad \lambda_{133} = 4.10\times 10^{-1}\,.
\end{aligned} \tag{97}$$

The second case is obtained by changing only the mixing terms as in the neutral sector. As a result, the masses are *the same* as in the previous case. Thus, the ordinary charged leptons get the consistent masses; and, the lightest chargino with the mass of 45.9104 GeV is in the experimental lower limit of 45 GeV. It was shown in this section that there are nine fermions. However, as mentioned above, by the conservation of $R$-parity, there are only six charginos.

To summarize, as above we have given at the tree level the consistent masses for the charged leptons and the neutrinos in the supersymmetric 3-3-1 model with right-handed neutrinos. Such a model for the leptons is simpler than that of the supersymmetric minimal 3-3-1 model [56], where the loop corrections are needed for the masses of the leptons [57]. The charginos and neutralinos in this model gain masses respectively very smaller than those of the supersymmetric minimal 3-3-1 model [57]. Contrasting with the supersymmetric minimal 3-3-1 model, the MSSM neutralinos and charginos in this model can be directly identified via the mass spectra given above, where the mass constraints on the MSSM particles can be found in [78].

## 5 Conclusion

In this article, we have found, in framework of the supersymmetric 3-3-1 model with right-handed neutrinos, definitions of the $R$-charges which are similar to that in the MSSM. This means that in the model considered, there is one discrete symmetry which allows the neutral and charged fermions, gauge bosons and scalar fields to get masses, at same time forbidding proton decay. Thus, in this case there exists a phenomenology similar to the MSSM with the famous missing transverse energy events, which is specific of the $R$-parity conservation [77].

We have shown that there is one symmetry which gives neutrinos masses but forbids proton decay. Unlike the cases with the MSSM and the minimal 3-3-1 model [57], in this model, all the fermions get masses at the tree level.

The famous relation for the $R$-parity in the MSSM has been generalized to this kind of 3-3-1 models. In this case it relates to the new conserved charge $\mathcal{L}$. A simple mechanism for the mass generation of the neutrinos has been explored. We have shown that the model naturally gives rise to an inverted hierarchy mass pattern for the neutrinos. Moreover, the MSSM superpartners in this model can be explicitly identified, which is unlike the case of the supersymmetric extension of the minimal 3-3-1 model.

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## Appendix: Mass matrix elements of $Y^0$

The nonzero elements of $Y^0$ are

$$\begin{aligned}
&m_{1,5} = m_{5,1} = -\frac{u}{3}(\lambda_{421} - \lambda_{412})\,,\\
&m_{1,6} = m_{6,1} = -\frac{u}{3}(\lambda_{431} - \lambda_{413})\,,\\
&m_{1,13} = m_{13,1} = \frac{\mu_{01}}{2}\,, \quad m_{1,14} = m_{14,1} = -\frac{\lambda_{31}}{3}u\,,\\
&m_{1,17} = m_{17,1} = \frac{\mu_{11}}{2}\,, \quad m_{1,18} = m_{18,1} = -\frac{\lambda_{21}}{3}u\,,\\
&m_{1,20} = m_{20,1} = -\frac{\lambda_{21}}{3}w\,,\\
&m_{2,4} = m_{4,2} = -\frac{u}{3}(\lambda_{412} - \lambda_{421})\,,\\
&m_{2,6} = m_{6,2} = -\frac{u}{3}(\lambda_{432} - \lambda_{423})\,, m_{2,13} = m_{13,2} = \frac{\mu_{02}}{2}\,,\\
&m_{2,14} = m_{14,2} = -\frac{\lambda_{32}}{3}u\,, \quad m_{2,17} = m_{17,2} = \frac{\mu_{12}}{2}\,,\\
&m_{2,18} = m_{18,2} = -\frac{\lambda_{22}}{3}u, \quad m_{2,20} = m_{20,2} = -\frac{\lambda_{22}}{3}w\,,\\
&m_{3,4} = m_{4,3} = -\frac{u}{3}(\lambda_{413} - \lambda_{431})\,,\\
&m_{3,5} = m_{5,3} = -\frac{u}{3}(\lambda_{423} - \lambda_{432})\,,\\
&m_{3,13} = m_{13,3} = \frac{\mu_{03}}{2}\,, \quad m_{3,14} = m_{14,3} = -\frac{\lambda_{33}}{3}u\,,\\
&m_{3,17} = m_{17,3} = \frac{\mu_{13}}{2}\,, \quad m_{3,18} = m_{18,3} = -\frac{\lambda_{23}}{3}u\,,\\
&m_{3,20} = m_{20,3} = -\frac{\lambda_{23}}{3}w\,, \quad m_{4,12} = m_{12,4} = \frac{\lambda_{31}}{3}u\,,\\
&m_{4,15} = m_{15,4} = \frac{\mu_{01}}{2}\,, \quad m_{4,16} = m_{16,4} = \frac{\lambda_{21}}{3}u\,,\\
&m_{4,19} = m_{19,4} = \frac{\mu_{11}}{2}\,, \quad m_{4,20} = m_{20,4} = \frac{\lambda_{31}}{3}v\,,\\
&m_{5,12} = m_{12,5} = \frac{\lambda_{32}}{3}u\,, \quad m_{5,15} = m_{15,5} = \frac{\mu_{02}}{2}\,,\\
&m_{5,16} = m_{16,5} = \frac{\lambda_{22}}{3}u\,, \quad m_{5,19} = m_{19,5} = \frac{\mu_{12}}{2}\,,\\
&m_{5,20} = m_{20,5} = \frac{\lambda_{32}}{3}v\,, \quad m_{6,12} = m_{12,6} = \frac{\lambda_{33}}{3}u\,,\\
&m_{6,15} = m_{15,6} = \frac{\mu_{03}}{2}\,, \quad m_{6,16} = m_{16,6} = \frac{\lambda_{23}}{3}u\,,\\
&m_{6,19} = m_{19,6} = \frac{\mu_{13}}{2}\,, \quad m_{6,20} = m_{20,6} = \frac{\lambda_{33}}{3}v\,,\\
&m_{7,7} = -m_\lambda\,, \quad m_{7,12} = m_{12,7} = \frac{gv}{\sqrt{2}}\,,\\
&m_{7,13} = m_{13,7} = -\frac{gv'}{\sqrt{2}}\,, \quad m_{7,20} = m_{20,7} = -\frac{gu}{\sqrt{2}}\,,\\
&m_{7,21} = m_{13,7} = -\frac{gu'}{\sqrt{2}}\,, \quad m_{8,9} = m_{9,8} = m_{10,10} = -m_\lambda\,,\\
&m_{8,14} = m_{14,8} = gv\,, \quad m_{8,17} = m_{17,8} = -gw'\,,\\
&m_{9,15} = m_{15,9} = -gv'\,, \quad m_{9,16} = m_{16,9} = gw\,,
\end{aligned}$$

$$
\begin{aligned}
&m_{10,12} = m_{12,10} = \frac{gv}{\sqrt{6}}\,, \; m_{10,13} = m_{13,10} = -\frac{gv'}{\sqrt{6}}\,,\\
&m_{10,18} = m_{18,10} = -\sqrt{\frac{2}{3}}gw\,, \; m_{10,19} = m_{19,10} = \sqrt{\frac{2}{3}}gw'\,,\\
&m_{10,20} = m_{20,10} = \frac{gu}{\sqrt{6}}\,, \; m_{10,21} = m_{21,10} = -\frac{gu'}{\sqrt{6}}\,,\\
&m_{11,11} = -m'\,, \; m_{11,12} = m_{12,11} = -\frac{g'v}{3\sqrt{2}}\,,\\
&m_{11,13} = m_{13,11} = \frac{g'v'}{3\sqrt{2}}\,, \; m_{11,18} = m_{18,11} = -\frac{g'w}{3\sqrt{2}}\,,\\
&m_{11,19} = m_{19,11} = \frac{g'w'}{3\sqrt{2}}\,, \; m_{11,20} = m_{20,11} = \sqrt{\frac{2}{3}}g'u\,,\\
&m_{11,21} = m_{21,11} = -\sqrt{\frac{2}{3}}g'u'\,, \; m_{12,13} = m_{13,12} = \frac{\mu_\eta}{2}\,,\\
&m_{12,17} = m_{17,12} = \frac{\mu_2}{2}\,, \; m_{12,18} = m_{18,12} = \frac{f_1}{3}u\,,\\
&m_{12,20} = m_{20,12} = \frac{f_1}{3}w\,, \; m_{13,16} = m_{16,13} = \frac{\mu_3}{2}\,,\\
&m_{13,19} = m_{19,13} = \frac{f_1'}{3}u'\,, \; m_{13,21} = m_{21,13} = \frac{f_1'}{3}w'\,,\\
&m_{14,15} = m_{15,14} = \frac{\mu_\eta}{2}\,, \; m_{14,16} = m_{16,14} = -\frac{f_1}{3}u\,,\\
&m_{14,19} = m_{19,14} = \frac{\mu_2}{2}\,, \; m_{15,17} = m_{17,15} = -\frac{f_1'}{3}u'\,,\\
&m_{15,18} = m_{18,15} = \frac{\mu_3}{2}\,, \; m_{16,17} = m_{17,16} = \frac{\mu_\chi}{2}\,,\\
&m_{17,19} = m_{19,17} = \frac{\mu_\chi}{2}\,, \; m_{17,20} = m_{20,17} = \frac{f_1}{3}u\,,\\
&m_{18,19} = m_{19,18} = \frac{\mu_\chi}{2}\,, \; m_{18,21} = m_{21,18} = \frac{f_1}{3}v\,,\\
&m_{19,21} = m_{21,19} = \frac{f_1'}{3}u'\,, \; m_{20,21} = m_{21,20} = \frac{\mu_\rho}{2}\,. \qquad \text{(A.1)}
\end{aligned}
$$

## References


1. Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **81**, 1562 (1998)
2. Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **82**, 2644 (1999)
3. Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **85**, 3999 (2000)
4. Y. Suzuki, Nucl. Phys. B Proc. Suppl. **77**, 35 (1999)
5. Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **81**, 1158 (1998)
6. Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **82**, 1810 (1999)
7. S. Fukuda et al., Phys. Rev. Lett. **86**, 5651 (2001)
8. H.Ts. Wong, in Proceedings of Hanoi Forum on Frontiers of Basic Science: Towards New Physics: Earth and Space Science, Mathematics, Hanoi, Vietnam, 27–29 September, 2005 (Osaka University Press, 2006), pp. 74–79 [arXiv:hep-ex/0510001]
9. G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, arXiv:hep-ph/0506083
10. P. Minkowski, Phys. Lett. B **67**, 421 (1977)
11. M. Gell-Mann, P. Ramond, R. Slansky, in: Supergravity, ed. by P. van Nieuwenhuizen et al. (North Holland, Amsterdam, 1980), p. 315
12. T. Yanagida, in: Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, ed. by O. Sawada, A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95
13. S.L. Glashow, The future of elementary particle physics, in Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons, ed. by M. Lévy et al. (Plenum Press, New York, 1980), p. 687
14. R.N. Mohapatra, G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980)
15. A. Zee, Phys. Lett. B **93**, 389 (1980)
16. A. Zee, Phys. Lett. B **161**, 141 (1985)
17. A. Zee, Nucl. Phys. B **264**, 99 (1986)
18. L. Wolfenstein, Nucl. Phys. B **175**, 93 (1980)
19. K.S. Babu, Phys. Lett. B **203**, 132 (1988)
20. K.S. Babu, E. Ma, Phys. Rev. Lett. **61**, 674 (1988)
21. D. Chang, W.-Y. Keung, P.B. Pal, Phys. Rev. Lett. **61**, 2420 (1988)
22. J.T. Peltonieri, A.Y. Smirnov, J.W. Valle, Phys. Lett. B **286**, 321 (1992)
23. D. Choodhury, R. Gandhi, J.A. Gracey, B. Mukhopadhyaya, Phys. Rev. D **50**, 3468 (1994)
24. D. Chang, A. Zee, Phys. Rev. D **61**, 071 303(R) (2000)
25. C. Jarlskog, M. Matsuda, S. Skalhauge, M. Tanimoto, Phys. Lett. B **449**, 240 (1999)
26. M. Frigerio, A.Y. Smirnov, JHEP **0302**, 004 (2003)
27. F. Pisano, V. Pleitez, Phys. Rev. D **46**, 410 (1992)
28. P.H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992)
29. R. Foot et al., Phys. Rev. D **47**, 4158 (1993)
30. M. Singer, J.W.F. Valle, J. Schechter, Phys. Rev. D **22**, 738 (1980)
31. R. Foot, H.N. Long, T. A. Tran, Phys. Rev. D **50**, 34(R) (1994)
32. J.C. Montero, F. Pisano, V. Pleitez, Phys. Rev. D **47**, 2918 (1993)
33. H.N. Long, Phys. Rev. D **53**, 437 (1996)
34. H.N. Long, Phys. Rev. D **54**, 4691 (1996)
35. F. Pisano, Mod. Phys. Lett. A **11**, 2639 (1996)
36. C.A. de S. Pires, O.P. Ravinez, Phys. Rev. D **58**, 035 008 (1998)
37. J.C. Montero, C.A. de S. Pires, V. Pleitez, Phys. Lett. B **502**, 167 (2001)
38. A.G. Dias, V. Pleitez, Phys. Rev. D **69**, 077 702 (2004)
39. A.G. Dias, A. Doff, Phys. Rev. D **72**, 035 006 (2005)
40. T. Kitabayashi, M. Yasuè, Phys. Rev. D **63**, 095 002 (2001)
41. T. Kitabayashi, M. Yasuè, Phys. Lett. B **508**, 85 (2001)
42. T. Kitabayashi, M. Yasuè, Phys. Rev. D **63**, 095 006 (2001)
43. T. Kitabayashi, M. Yasuè, Nucl. Phys. B **609**, 61 (2001)
44. T. Kitabayashi, M. Yasuè, Phys. Rev. D **67**, 015 006 (2003)
45. Y. Okamoto, M. Yasuè, Phys. Lett. B **466**, 267 (1999)
46. J.C. Montero, C.A. de S. Pires, V. Pleitez, Phys. Rev. D **65**, 095 001 (2002)
47. A.A. Gusso, C.A. de S. Pires, P.S. Rodrigues da Silva, Mod. Phys. Lett. A **18**, 1849 (2003)
48. I. Aizawa et al., Phys. Rev. D **70**, 015 011 (2004)
49. H.N. Long, T. Inami, Phys. Rev. D **61**, 075 002 (2000)
50. P.V. Dong, H.N. Long, D.T. Nhung, D.V. Soa, Phys. Rev. D **73**, 035 004 (2006)
51. P.V. Dong, H.N. Long, D.V. Soa, Phys. Rev. D **73**, 075 005 (2006)

52. D. Chang, H.N. Long, Phys. Rev. D **73**, 053 006 (2006)
53. A.G. Dias, C.A. de S. Pires, P.S. Rodrigues da Silva, Phys. Lett. B **628**, 85 (2005)
54. J.C. Montero, V. Pleitez, M.C. Rodriguez, Phys. Rev. D **70**, 075 004 (2004)
55. D.T. Huong, M.C. Rodriguez, H.N. Long, arXiv:hep-ph/0508045
56. J.C. Montero, V. Pleitez, M.C. Rodriguez, Phys. Rev. D **65**, 035 006 (2002)
57. J.C. Montero, V. Pleitez, M.C. Rodriguez, Phys. Rev. D **65**, 095 008 (2002)
58. A. Salam, J. Strathdee, Nucl. Phys. B **87**, 85 (1975)
59. P. Fayet, Nucl. Phys. B **90**, 104 (1975)
60. R. Barbier et al., Phys. Rep. **420**, 1 (2005) [arXiv:hep-ph/0406039]
61. G. Moreau, arXiv:hep-ph/0012156
62. D. Bailin, A. Love, Supersymmetric Gauge Field Theory and String Theory, 1st edition (Institute of Physics Publishing, Bristol UK, 1994)
63. J. Wess, J. Bagger, Supersymmetry and Supergravity, 2nd edition (Princeton University Press, Princeton NJ, 1992)
64. H.E. Haber, G.L. Kane, Phys. Rep. **117**, 75 (1985)
65. L. Girardello, M.T. Grisaru, Nucl. Phys. B **194**, 65 (1982)
66. C. Aulakh, R.N. Mohapatra, Phys. Lett. B **119**, 136 (1982)
67. R. Mohapatra, Phys. Rev. D **34**, 3457 (1986)
68. P. Fayet, New Frontiers in High-Energy Physics, in: Proceedings Orbis Scientiae, Coral Gables, Florida, USA, ed. by A. Perlmutter, L.F. Scott (Plenum, New York, 1978), p. 413
69. G.R. Farrar, P. Fayet, Phys. Lett. B **76**, 575 (1978)
70. L. Hall, M. Suzuki, Nucl. Phys. B **231**, 419 (1984)
71. L.E. Ibañez, G.G. Ross, Nucl. Phys. B **368**, 3 (1992)
72. L.J. Hall, M. Suzuki, Nucl. Phys. B **231**, 419 (1984)
73. T. Banks, Y. Grossman, E. Nardi, Y. Nir, Phys. Rev. D **52**, 5319 (1995)
74. M.A. Diaz, J.C. Romão, J.W.F. Valle, Nucl. Phys. B **524**, 23 (1998)
75. Particle Data Group, S. Eidelman et al., Phys. Lett. B **592**, 1 (2004)
76. M. Capdequi-Peyranère, M.C. Rodriguez, Phys. Rev. D **65**, 035 006 (2002)
77. M. Drees, R. Godbole, P. Roy, Theory and Phenomenology of Sparticles (World Scientific Publishing, Singapore, 2004)
78. L. Roszkowski, arXiv:hep-ph/9302259